2020
EDITION

THREE DIMENSIONAL GEOMETRY

KCSE PAST QUESTIONS ON THE TOPIC AND ANSWERS

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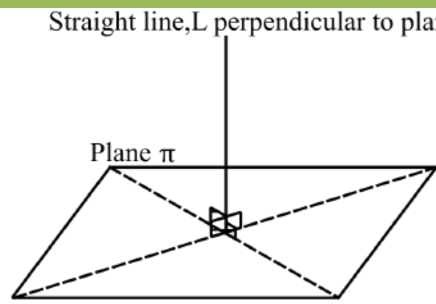
THREE DIMENSIONAL GEOMETRY

Basic concepts.

- A figure with volume is said to be in three dimensions.
- **Line perpendicular to a plane**

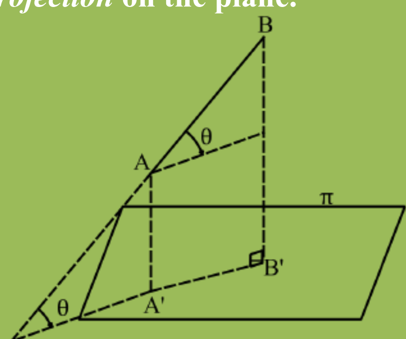
A straight line is said to be perpendicular to a plane if it is perpendicular to any straight line on the plane.



- **Angle between a straight line and a plane.**

If AB is a straight line and A' and B' are two points on a plane π such that AA' and BB' are perpendicular to the plane,then $A'B'$ is known as the projection of AB on the plane{the shadow of line AB}.Try question 28 of this document.

The angle between straight line and a plane is defined as the angle angle(θ) between the line and its *projection* on the plane.



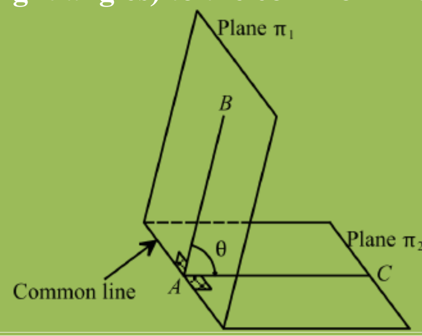
Line: AB
Plane: π
Normal to plane: BB'
Projection on π: $A'B'$
Angle between AB and the plane π is θ

The angle between a straight line and a plane is also known as the angle of inclination of the line to the plane.

- **Angle between two planes.**

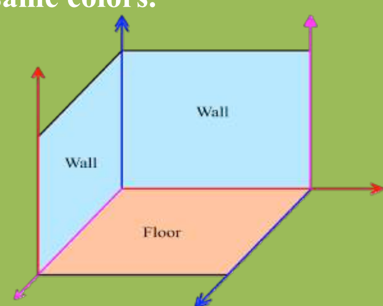
Two planes π_1 and π_2 **,which are not parallel have a line of intersection called the** common line.

The angle between two planes is the angle(θ) **between any two lines** AB and AC **,where** A **is a point on the common line on one plane,and the lines** AB **on the plane and** AC **on the other plane are both perpendicular(right angles) to the common line.**



- **Angle between two skew lines.**

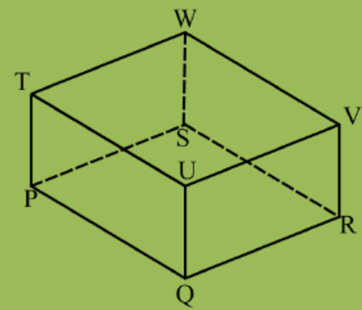
Skew lines are type of non intersecting lines in space/lie in different planes.
You can identify skew lines when you look at the floor of your room as shown below.
The edges formed by the walls with the floor form skew lines with the intersecting lines of the walls as shown in diagram.
The skew lines are shown with same colors.



The angle between any two skew lines can found by translating one line such that its image intersects the other line.

Example;

Given that PQ = SR = TU = 10 cm,TP = UQ = WS = VR = 8 cm and QR = PS = UV = TW = 15 cm.

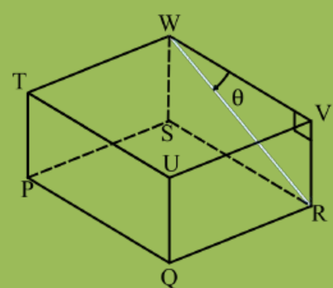


Calculate the angle between the skew lines:

(a) PQ and WR

Solution

Translate PQ to WV to get the angle between PQ and WR i.e. (θ)



$$\tan\theta = \frac{VR}{WV} = \frac{8}{10}$$

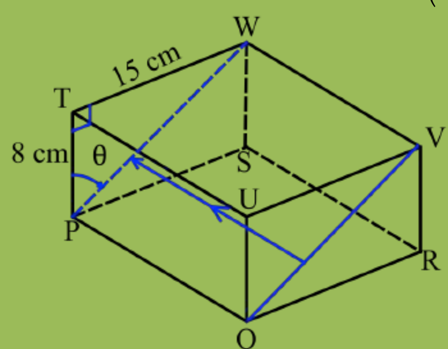
$$\theta = \tan^{-1}\left(\frac{8}{10}\right)$$

$$= 38\cdot 66^0$$

(b) QV and PTS

Solution

Translate PQ to PW . (θ) is the angle between QV and PT



$$\tan\theta = \frac{TW}{TP} = \frac{15}{8} = 1\cdot 875$$

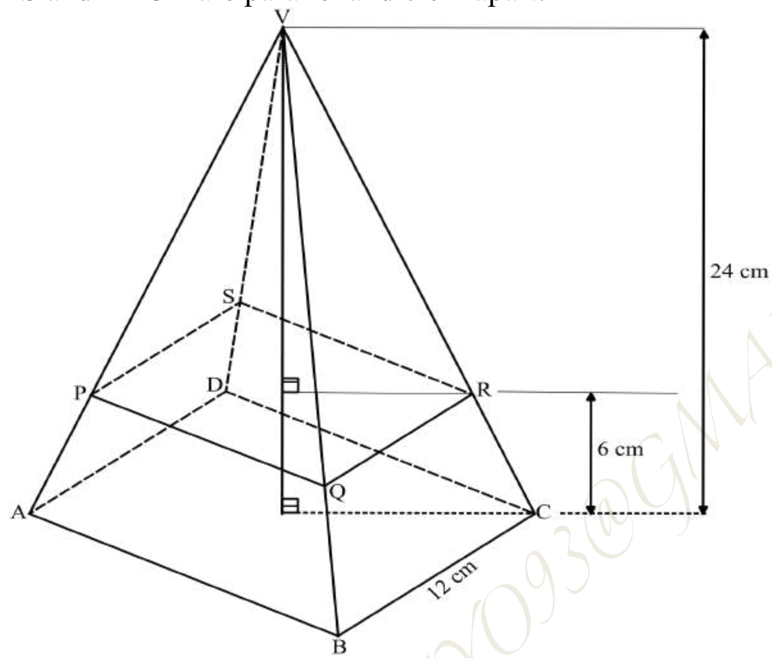
$$\theta = \tan^{-1}(1\cdot 875)$$

$$= 61\cdot 93^0$$

QUESTIONS

1. **1990 Paper 2 Number 23**

The figure below shows a right pyramid VABCD whose rectangular base is 18 cm by 12 cm.The altitude is 24 cm.The plane PQRS and ABCD are parallel and 6 cm apart.



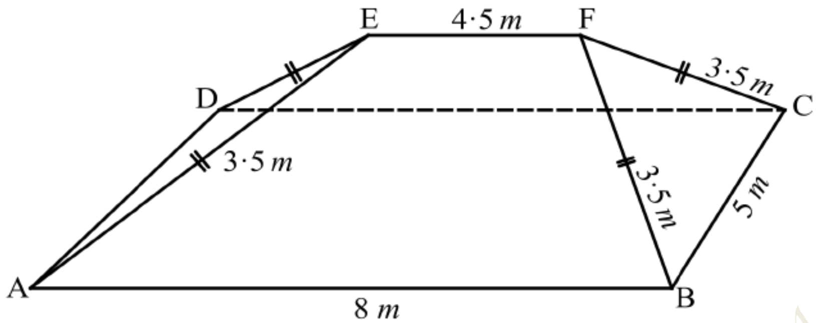
Calculate;

(a) the angle between the planes ABCD and VAB. (4 marks)

(b) the area of the rectangle PQRS. (4 marks)

2. **1991 Paper 2 Number 23**

The figure below shows a shape of a roof with a horizontal rectangular base ABCD.The ridge EF is also horizontal. The measurements of the roof are AB = 8 m,BC = 5 m,EF = 4·5 m and EA = ED = FC = 3·5 m



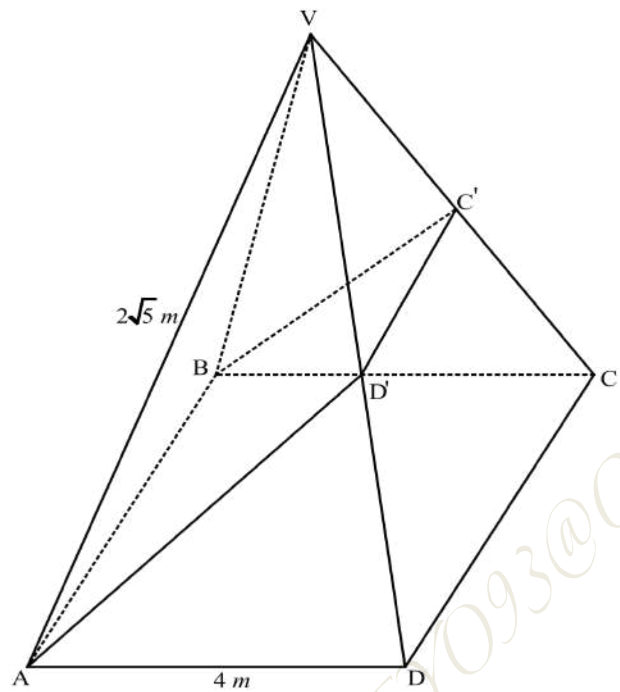
Calculate:

(i) The height of the ridge EF above the base ABCD. (4 marks)

(ii) The angle between the face AED and the base ABCD. (4 marks)

3. **1992 Paper 2 Number 19**

A right pyramid VABCD has a square base ABCD of sides 4 m.The slant edges VA,VB,VC and VD are $2\sqrt{5}$ m long.



(a) Calculate;

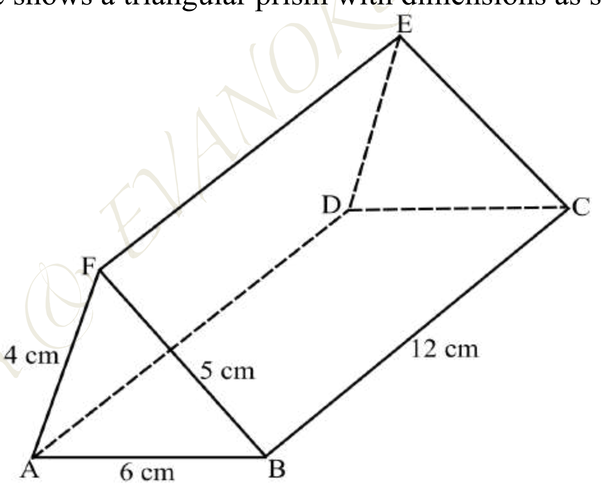
(i) the height of the pyramid. (3 marks)

(ii) the angle between the plane VAB and the base ABCD. (2 marks)

(b) C' and D' are midpoints of VC and VD respectively. Calculate the angle between the planes ABCD and $ABC'D'$ (3 marks)

4. **1993 paper 2 number 22.**

The figure on the next page shows a triangular prism with dimensions as shown.



Calculate:

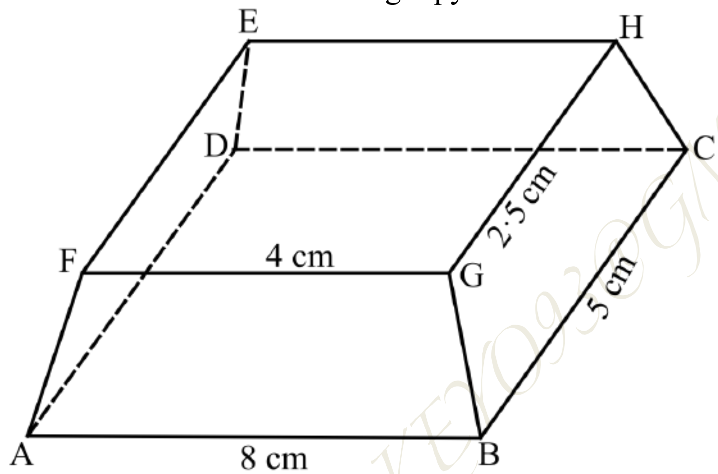
(a) the angle between the faces FBCE and ABCD. (2 marks)

(b) the volume of the prism. (3 marks)

(c) the angle between the planes DFC and ABCD. (3 marks)

5. 1994 paper 1 number 22

In the figure below ABCDEFGH is a frustum of a right pyramid.The altitude of the frustum is 2 cm.



Calculate:

(a) the altitude of the pyramid. (2 marks)

(b) the volume of the frustum. (3 marks)

(c) the angle between the base of the frustum and the face ABGF. (3 marks)

6. **1996 paper 2 number 13**

The base of a right pyramid is a square ABCD of side $2a$ cm .The slant edges VA,VB,VC and VD are each of length $3a$ cm .

(a) Sketch and label the pyramid. (1 mark)

(b) Find the angle between a slanting edge and the base. (2 marks)

7. **1997 paper 2 number 6**

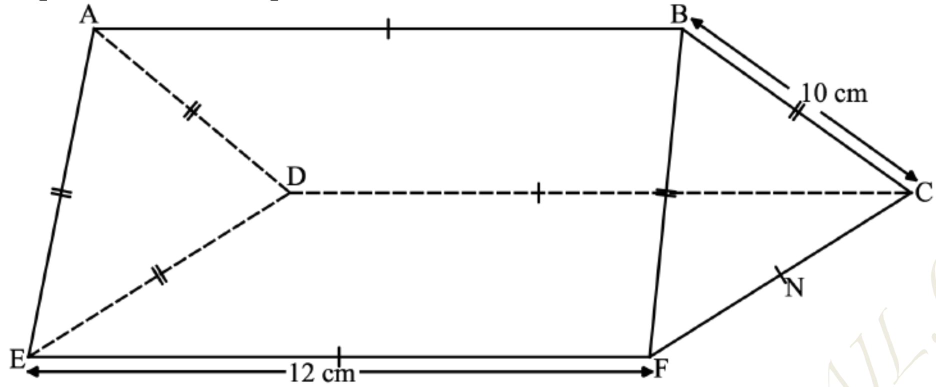
A pyramid of height 10 cm stands on a square base ABCD of side 6 cm.

(a) Draw a sketch of the pyramid. (1 mark)

(b) Calculate the perpendicular distance from the vertex to the side AB. (2 marks)

8. **1998 paper 2 number 16**

The triangular prism shown below has the sides AB = DC = EF = 12 cm. The ends are equilateral triangles of sides 10 cm. The point N is the midpoint of FC.



(a) Find the length of:

(i) BN (1 mark)

(ii) EN (1 mark)

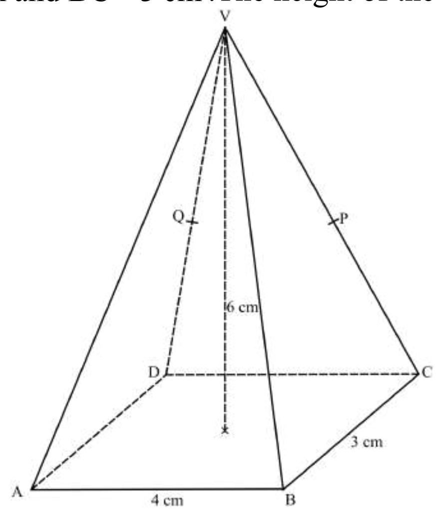
(b) Find the angle between the line EB and the plane CDEF. (2 marks)

9. **1999 paper 1 number 14**

An equilateral triangle ABC lies in a horizontal plane. A vertical flag AH stands at A. If AB = 2AH, find the angle between the planes ABC and HBC. (3 marks)

10. 1999 paper 2 number 24

The diagram below shows a right pyramid VABCD with V as the vertex. The base of the pyramid is a rectangle ABCD with AB = 4 cm and BC = 3 cm .The height of the pyramid is 6 cm.



(a) Calculate the;

(i) length of the projection of VA on the base. (2 marks)

(ii) angle between the face VAB and the base. (2 marks)

(b) P is the midpoint of VC and Q is the midpoint of VD.Find the angle between the planes VAB and the plane ABPQ. (4 marks)

11. **2000 paper 1 number 11.**

A pyramid VABCD has a rectangular horizontal base ABCD with AB = 12 cm and BC = 9 cm.The vertex V is vertically above A and VA = 6 cm. Calculate the volume of the pyramid. (2 marks)

12. **2001 paper 2 number 20.**

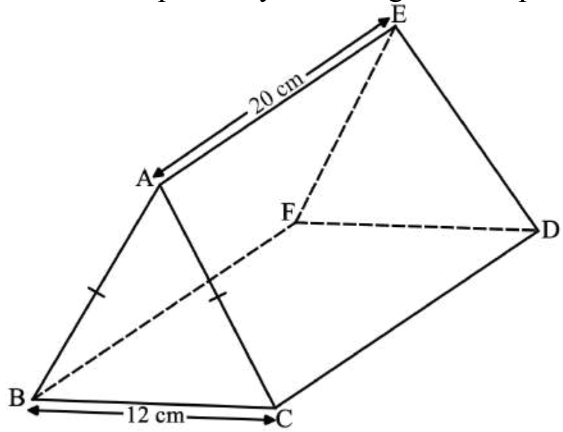
An electric pylon is 30 m high. A point S on top of the pylon is vertically above another point R on the ground. Points A and B are on the same horizontal ground as R.Point A is due south of the pylon and the angle of elevation of S from A is 26º.Point B is due west of the pylon and the angle of elevation of S from B is 32º.Calculate:

(a) distance from A to B. (6 marks)

(b) bearing of B from A. (2 marks)

13. 2002 paper 1 number 18.

The figure below represents a right prism whose triangular faces are isosceles. The base and height of each triangular face are 12 cm and 8 cm respectively. The length of the prism is 20 cm.



Calculate the:

(a) length CE. (3 marks)

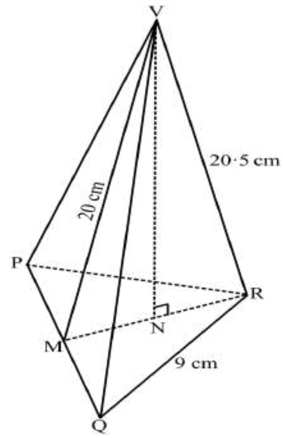
(b) angle between

(i) The line CE and the plane BCDF. (3 marks)

(ii) The plane EBC and the base BCDF. (2 marks)

14. 2002 paper 2 number 20.

The figure below VPQR below represents a model of a top of a tower. The horizontal base PQR is an equilateral triangle of side 9 cm.The lengths of the edges are VP = VQ = VR=20·5 cm .Point M is the mid point of PQ and VM = 20 cm .Point N is on the base and vertically below V.



Calculate the:

(a) (i) length of RM. (2 marks)

(ii) Height of the model. (2 marks)

(iii) Volume of the model. (2 marks)

(b) The model is made of material whose density is 2,700 kg/m^3. Find the mass of the model. (2 marks)

15. 2003 paper 1 number 15

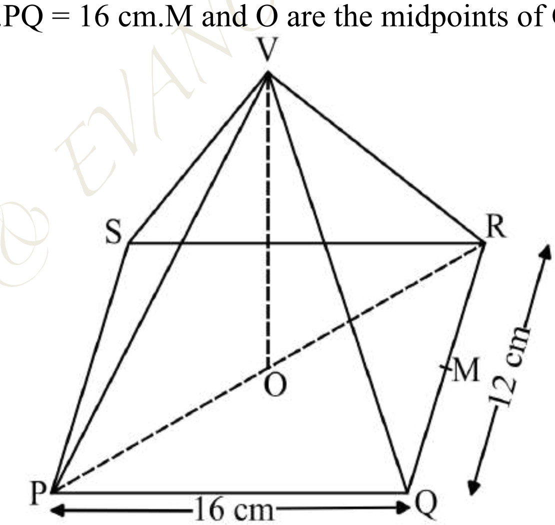
The points O,A and B are on the same horizontal ground. Point A is 80 metres to north of O.Point B is located 70 metres on a bearing of 060^0 from A. A vertical mast stands at point B.The angle of elevation of the top of the mass from O is 20^0 . Calculate:

(a) The distance of B from O. (2 marks)

(b) The height of the mast in metres. (2 marks)

16. 2003 paper 2 number 24

The figure below represents a right pyramid with vertex V and a rectangular base PQRS.
VP = VQ = VR = VS = 18 cm.PQ = 16 cm.M and O are the midpoints of QR and PR respectively.



Find:

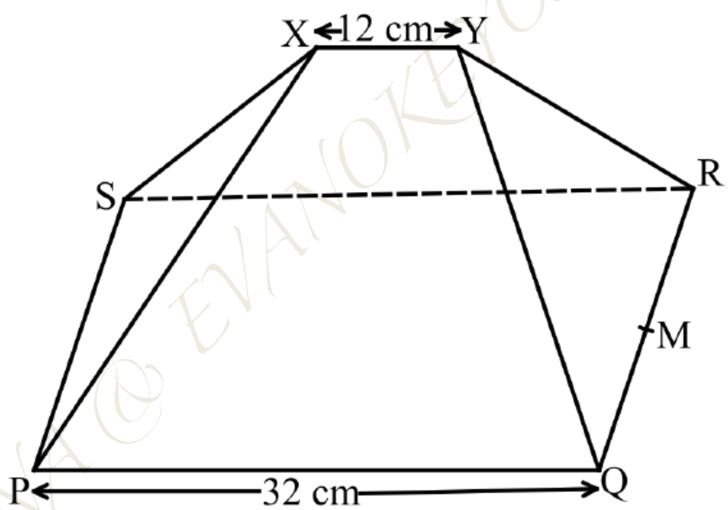
(a) the length of the projection of line VP on the plane PQRS. (2 marks)

(b) the size of the angle between the line VP and the plane PQRS. (2 marks)

(c) the size of the angle between the planes VDR and PQRS. (4 marks)

17. 2004 paper 2 number 24

The figure below shows a model of a roof with a rectangular base PQRS.PQ = 32 cm and QR = 14 cm. The ridge XY = 12 cm and is centrally placed.The faces PSX and QRY are equilateral triangles.M is the midpoint of QR.



Calculate:

(a) (i) the length of YM. (1 mark)

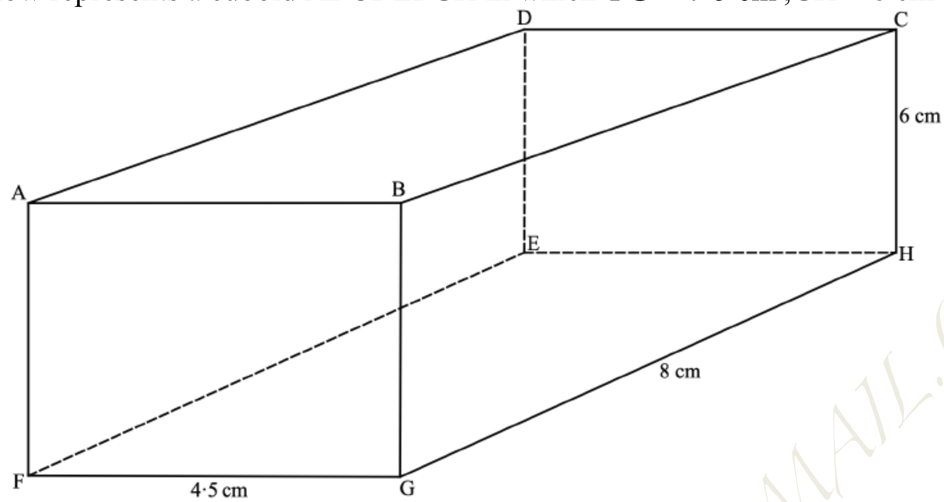
(ii) the height of Y above the base PQRS. (2 marks)

(b) the angle between the planes RSXY and PQRS. (3 marks)

(c) the acute angle between the lines XY and QS. (2 marks)

18. **2005 paper 2 number 23**

The diagram below represents a cuboid ABCDEFGH in which FG = 4·5 cm ,GH = 8 cm and HC = 6 cm.



Calculate:

(a) the length of FC. (2 marks)

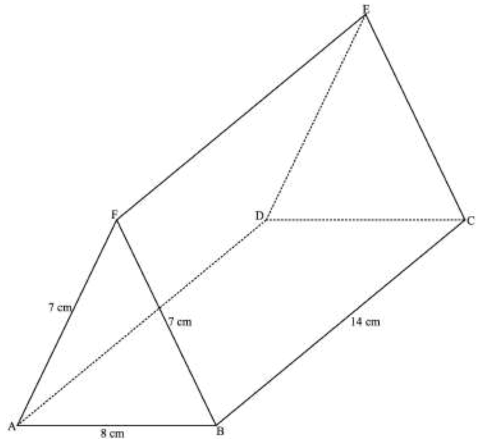
(b) (i) the size of the angle between the lines FC and FH. (2 marks)

(ii) the size of the angle between the lines AB and FH. (2 marks)

(c) the size of the angle between the plane ABHE and the plane FGHE. (2 marks)

19. 2008 paper 2 number 14

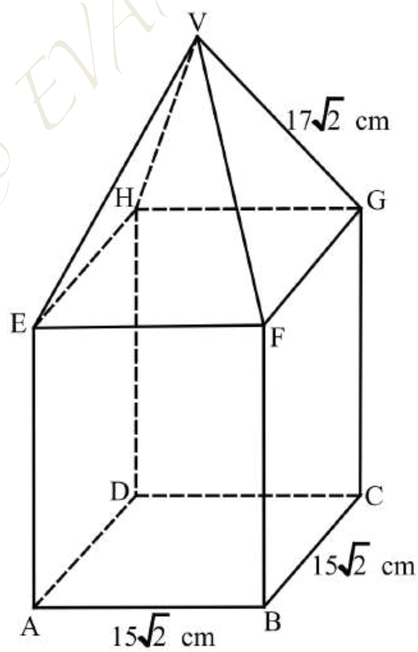
The figure below represents a triangular prism. The faces ABCD,ADEF and CBFE are rectangles. AB = 8 cm,BC = 14 cm,BF = 7 cm and AF = 7 cm.



Calculate the angle between faces BCEF and ABCD. (3 marks)

20. 2009 paper 2 number 22

The figure below shows a right pyramid mounted onto a cuboid. AB = BC = $15\sqrt{2}$ cm. CG = 8 cm and VG = $17\sqrt{2}$ cm



Calculate:

(a) the length AC; (1 mark)

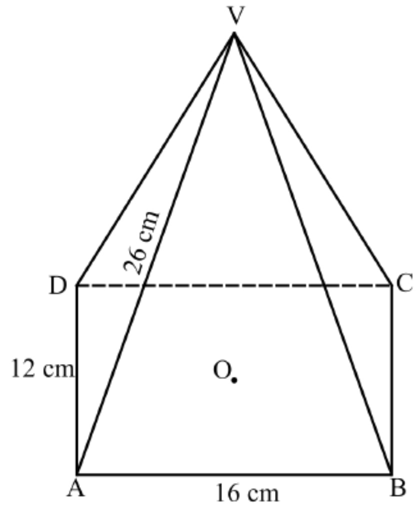
(b) the angle between the line AG and the plane ABCD; (3 marks)

(c) the vertical height of point V from the plane ABCD; (3 marks)

(d) the angle between the planes EFV and ABCD. (3 marks)

21. **2011 paper 2 number 22**

The figure below represents a rectangular based pyramid VABCD.AB = 12 cm and AD = 16 cm.Point O is vertically below V and VA = 26 cm.



Calculate:

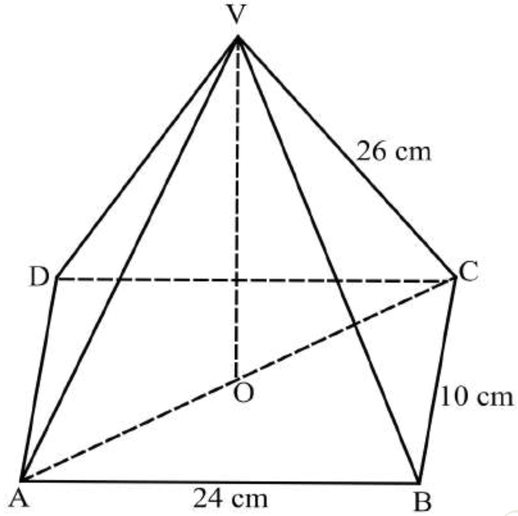
(a) the height,VO,of the pyramid; (4 marks)

(b) the angle between the edge VA and the plane ABCD; (3 marks)

(c) the angle between the planes VAB and ABCD. (3 marks)

22. 2012 paper 2 number 16

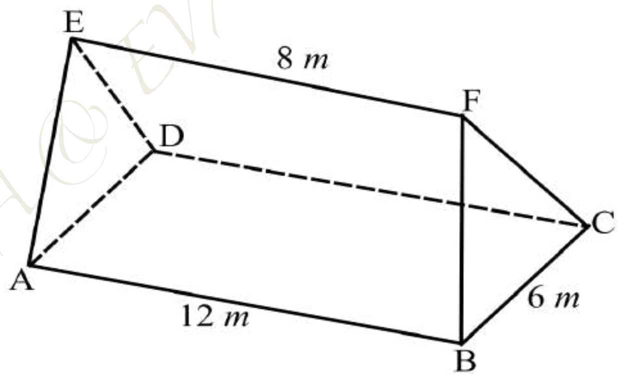
In the figure below,VABCD is a right pyramid on a rectangular base. Point O is vertically below the vertex V.AB = 24 cm,BC = 10 cm and CV = 26 cm.



Calculate the angle between the edge CV and the base ABCD. (3 marks)

23. 2013 paper 2 number 20

The figure ABCDEF below represents a roof of a house.AB = DC = 12 m,BC = AD = 6 m, AE = BF = CF = DE = 5 cm and EF = 8 m.



(a) Calculate correct to 2 decimal places, the perpendicular distance of EF from the plane ABCD. (3 marks)

(b) Calculate the angle between:

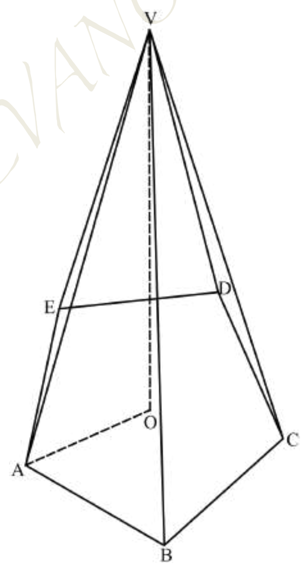
(i) the planes ADE and ABCD; (2 marks)

(ii) the line AE and the plane ABCD,correct to 1 decimal place; (2 marks)

(iii) the planes ABFE and DCFE,correct to 1 decimal place. (3 marks)

24. 2014 paper 1 number 20

The figure below shows a right pyramid VABCDE.The base ABCDE is regular pentagon.AO = 15 cm and VO = 36 cm.



Calculate:

(a) the area of the base correct to 2 decimal places; (3 marks)

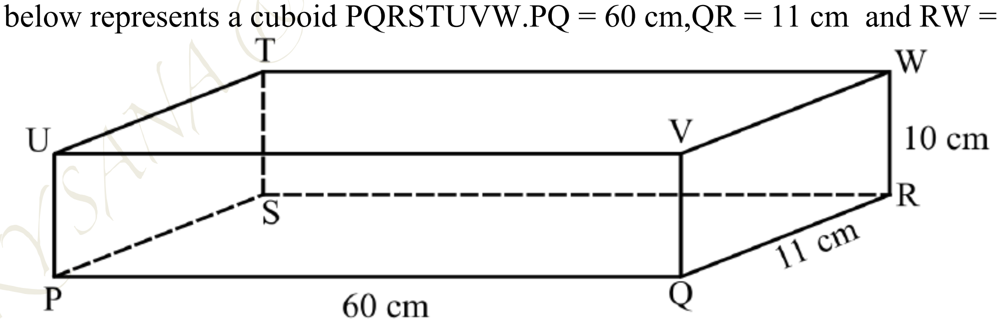
(b) the length AV; (1 mark)

(c) the surface are of the pyramid correct to 2 decimal places; (4 marks)

(d) the volume of the pyramid correct to 4 significant figures. (2 marks)

25. 2014 paper 2 number 10

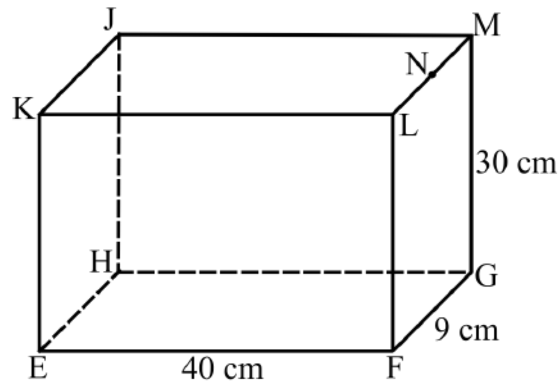
The figure below represents a cuboid PQRSTUVW.PQ = 60 cm,QR = 11 cm and RW = 10 cm.



Calculate the angle between the line PW and plane PQRS, correct to 2 decimal places. (3 marks)

26. 2015 paper 2 number 20

The figure below represents a cuboid EFGHJKLM in which EF = 40 cm,FG = 9 cm,GM = 30 cm.N is the midpoint of LM.



Calculate correct to 4 significant figures:

(a) the length of GL; (1 mark)

(b) the length of FJ: (2 marks)

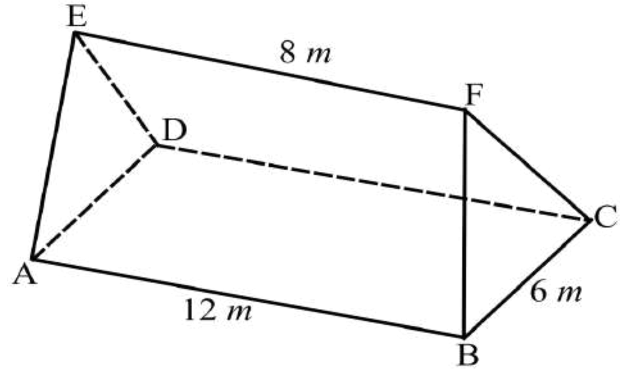
(c) the angle between EM and the plane EFGH; (2 marks)

(d) the angle between the planes EFGH and ENH; (2 marks)

(e) the angle between the lines EH and GL. (2 marks)

27. 2016 paper 2 number 19

The figure ABCDEF below represents a roof of a house.AB = DC = 12 m,BC = AD = 6 m, AE = BF = CF = DE = 5 cm and EF = 8 m.



(b) Calculate correct to 2 decimal places, the perpendicular distance of EF from the plane ABCD. (3 marks)

(b) Calculate the angle between:

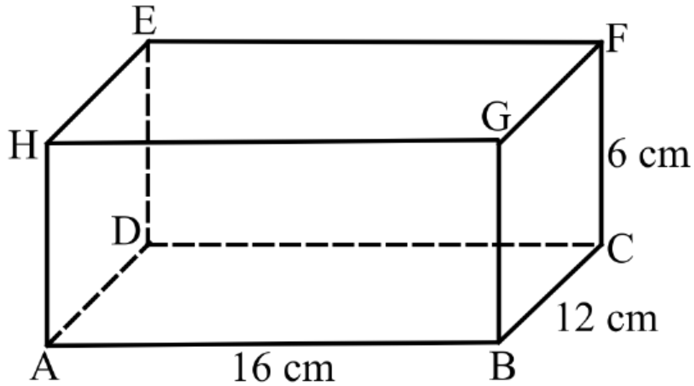
(i) the planes ADE and ABCD; (2 marks)

(ii) the line AE and the plane ABCD,correct to 1 decimal place; (2 marks)

(iii) the planes ABFE and DCFE,correct to 1 decimal place. (3 marks)

28. **2017 paper 2 number 20.**

The figure below represents a cuboid ABCDEFGH in which AB = 16 cm,BC = 12 cm and CF = 6 cm.



(a) Name the projection of the line BE on the plane ABCD. (1 mark)

(b) Calculate, correct to 1 decimal place:

(i) the size of the angle between AD and BF; (2 marks)

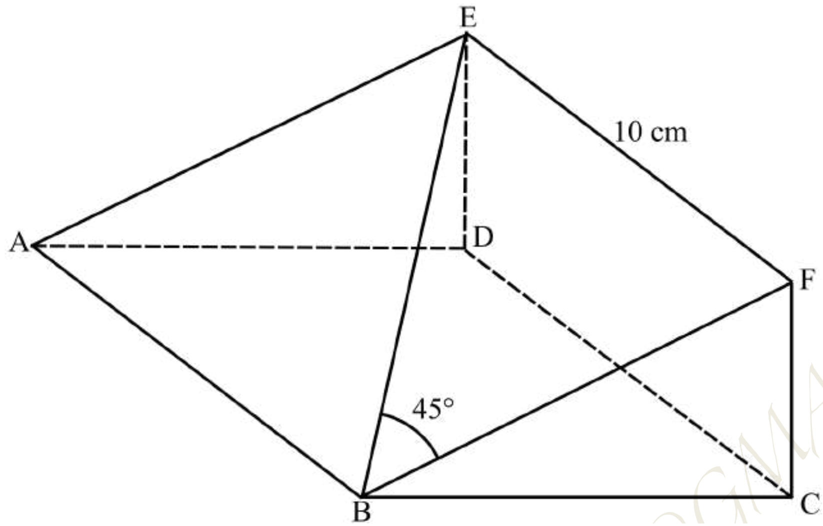
(ii) the angle between line BE and the plane ABCD; (3 marks)

(iii) the angle between planes HBCE and BCFG. (2 marks)

(c) Point N is the midpoint of EF.Calculate the length of BN,correct to a decimal place. (2 marks)

29. **2018 paper 2 number 4**

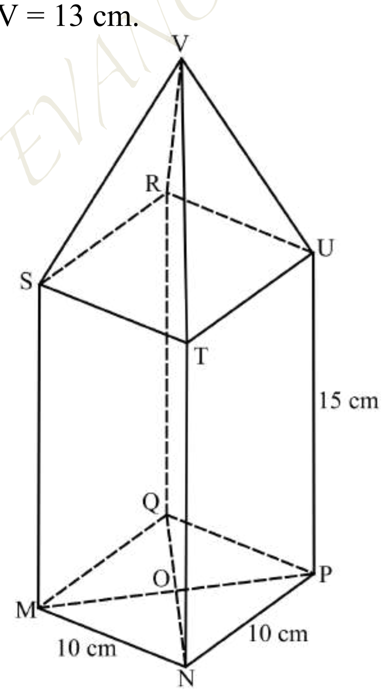
The figure below represents a wedge ABCDEF. EF 10 cm, angle FBE 45° and the angle between the planes ABFE and ABCD is 20°.



Calculate length BC, correct to l decimal place. (3 marks)

30. **2018 paper 2 number 22**

The figure below is a model of a watch tower with a square base of sides 10 cm. Height PU is 15 cm and slanting edges UV = TV = SV = RV = 13 cm.



Giving the answer correct to two decimal places, calculate:

(a) length MP; (2 marks)

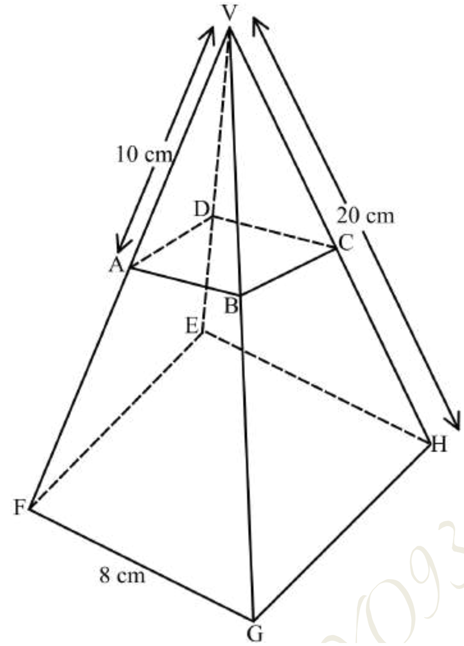
(b) the angle between MU and plane MNPQ; (2 marks)

(c) length of VO; (3 marks)

(d) the angle between planes VST and RSTU. (3 marks)

31. 2019 paper 1 number 20.

The figure below is a right pyramid VEFGHI with a square base of 8 cm and a slant edge of 20 cm. Points A,B,C and D lie on the slant edges of the pyramid such that VA = VB = VC = VD = I0 cm and plane ABCD is parallel to the base EFGH.



(a) Find the length of AB. (2 marks)

(b) Calculate, correct to 2 decimal places:

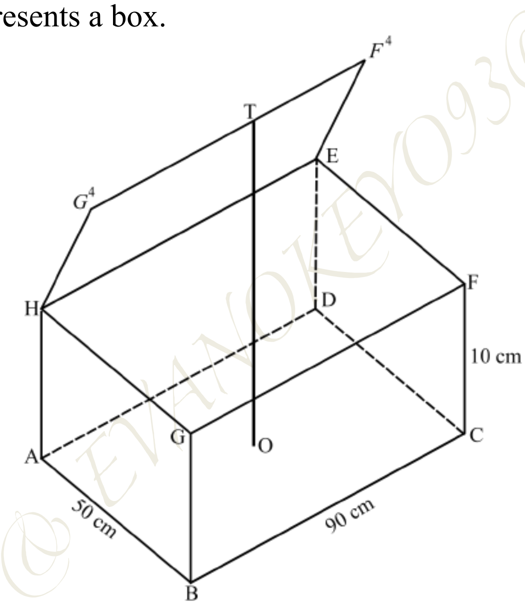
(i) The length of AC; (2 marks)

(ii) The perpendicular height of the pyramid VABCD (2 marks)

(c) The pyramid VABCD was cut off. Find the volume of the frustum ABCDEFGH correct to 2 decimal places. (4 marks)

32. **2019 paper 2 number 11.**

The figure ABCDEFGH represents a box.



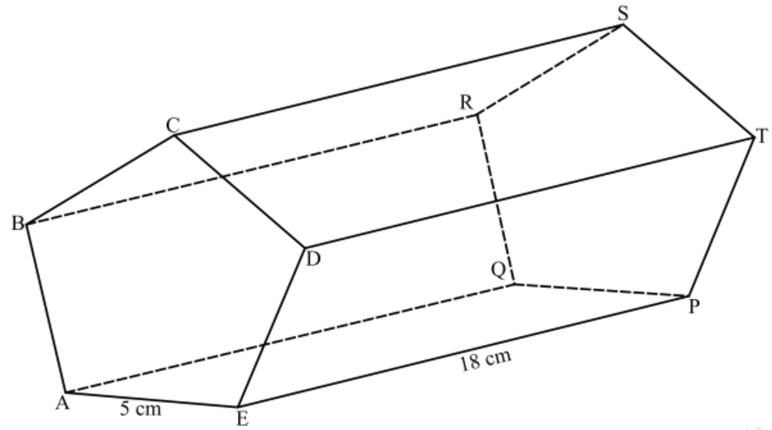
The top lid of the box is opened such that the height OT is 35 cm. Calculate the:

(a) angle the top lid makes with the plane FGHE; (2 marks)

(b) length BE, correct to 2 decimal places. (2 marks)

33. **Untested mode 1**

The figure below shows a prism whose cross – section is regular pentagon of side 5 cm.The length of the prism is 18 cm.



Calculate the angle between the planes:

(a) AEPQ and ADTQ. (3 marks)

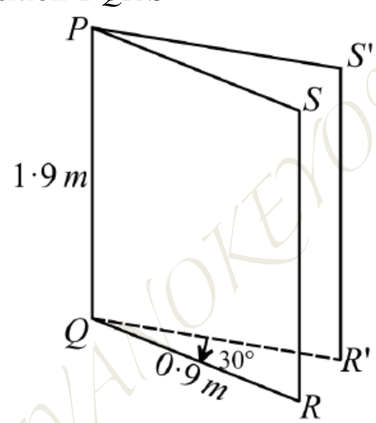
(b) AEPQ and ACSQ (3 marks)

(c) ACSQ and ADTQ (2 marks)

(d) ECSP and BDTR (2 marks)

34. Untested model 2.

The figure below shows a rectangular door with $PQ = 1\cdot 9\ m$ and $QR = 0\cdot 9m$ opened through 30º about the vertical line of hinges PQ to the position $PQR'S'$



Calculate, correct to two decimal places;

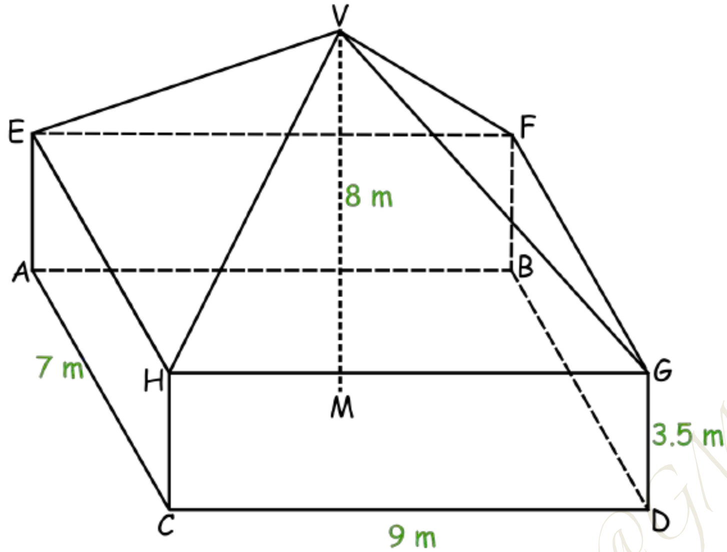
(a) the length of SQ ; (2 marks)

(b) the length of SS'; (2 marks)

(c) the $\angle SQS'$. (3 marks)

35. Untested model 3.

The diagram below shows a solid made of a cuboid and a pyramid. The apex of the pyramid V is directly above the centre O of ABCD.



Calculate the:

(a) angle between the line DV and the plane ABCD; (2 marks)

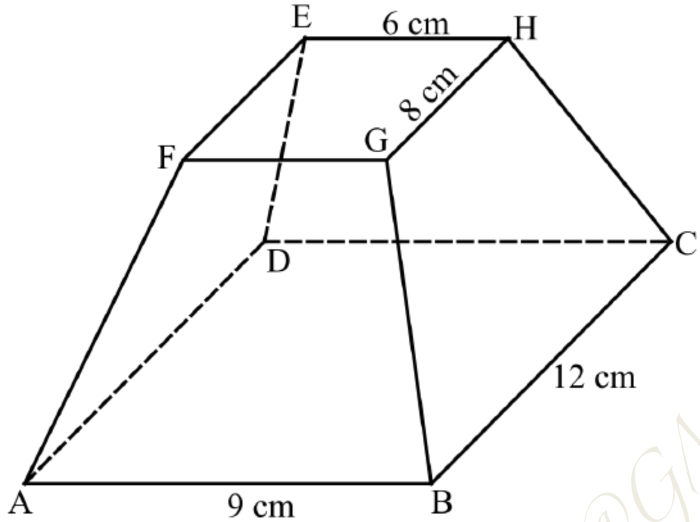
(b) angle between planes EHV and ACHE. (2 marks)

(c) volume of the pyramid of the solid. (3 marks)

(d) total surface area of the cuboid and the pyramid. (3 marks)

36. Untested model 4.

The figure below a frustum ABCDEFGH a right pyramid.AB = 9 cm,BC = 12 cm,FG = 6 cm,GH = 8 cm and the height of the frustum is 10 cm.



Calculate;

(a) the height of the pyramid ; (2 marks)

(b) the length of;

(i) AC (2 marks)

(ii) AH

(c) the angle between;

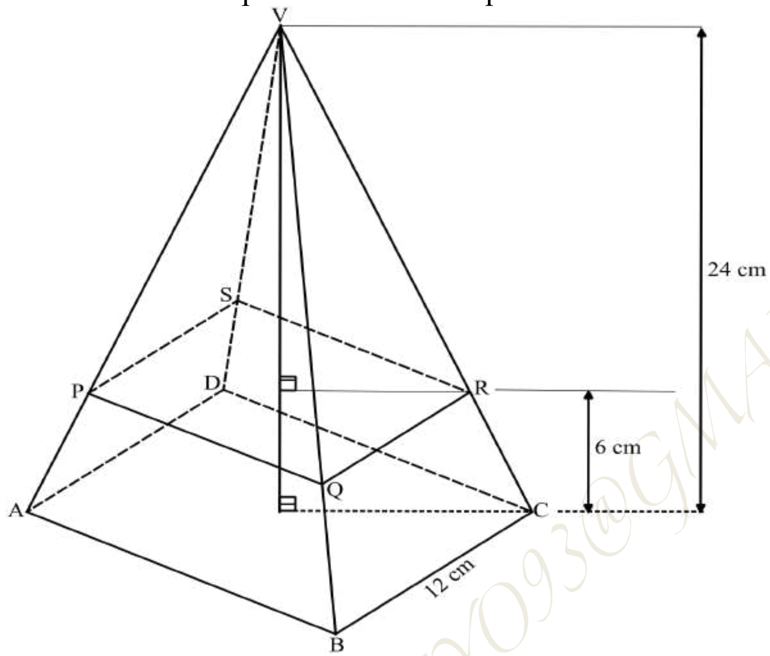
(i) line AH and the plane ABCD;

(ii) the planes ABHE and ABCD.

QUESTIONS AND ANSWERS

1. 1990 Paper 2 Number 23

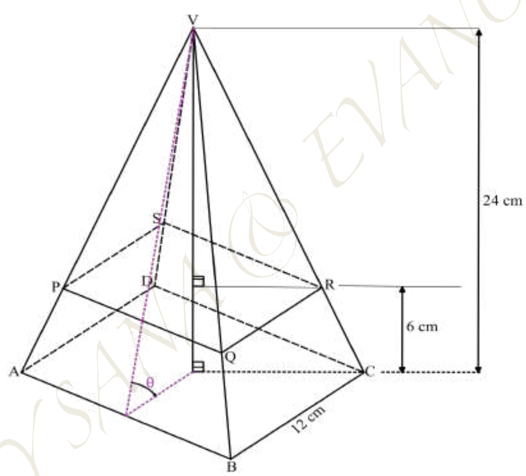
The figure below shows a right pyramid VABCD whose rectangular base is 18 cm by 12 cm.The altitude is 24 cm.The plane PQRS and ABCD are parallel and 6 cm apart.



Calculate;

(a) the angle between the planes ABCD and VAB. (4 marks)

Solution



$$\tan\theta = \frac{24}{\frac{1}{2}(12)}$$

$$\theta = \tan^{-1}\left(\frac{24}{6}\right)$$

$$= 75\cdot 96^0\ (4\ s.f)$$

(b) the area of the rectangle PQRS. (4 marks)

Solution

By similarity and enlargement;

$$\frac{PQ}{18} = \frac{24-6}{24}$$

$$PQ = \left(\frac{24-6}{24}\right)\times 18$$

$$= 13\cdot 5\text{ cm}$$

$$\frac{QR}{12} = \frac{18}{24}$$

$$QR = \left(\frac{18}{24}\right)\times 12$$

$$= 9\text{ cm}$$

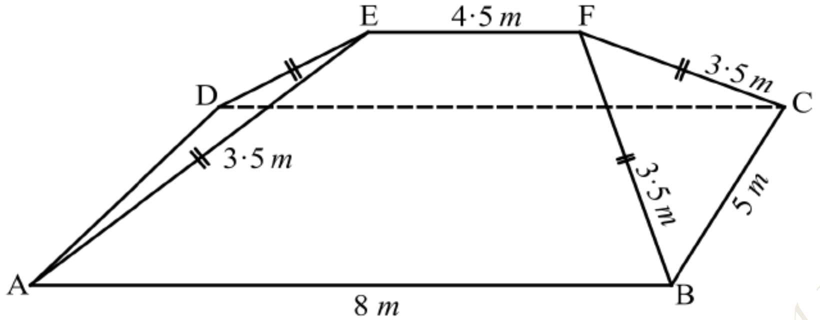
$$\text{Area} = L\times W$$

$$= 13\cdot 5\times 9$$

$$= 121\cdot 5\text{ cm}^2$$

2. **1991 Paper 2 Number 23**

The figure below shows a shape of a roof with a horizontal rectangular base ABCD.The ridge EF is also horizontal. The measurements of the roof are $AB = 8$ m, $BC = 5$ m, $EF = 4\cdot5$ m and $EA = ED = FC = 3\cdot5$ m



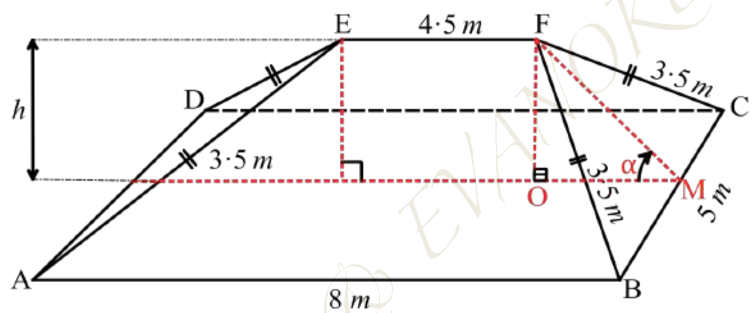
Calculate:

(i) The height of the ridge EF above the base ABCD. (4 marks)

Solution

→ *Slide the height / altitude to touch any of the surface / plane that you can easily view and join.*

→ *Produce perpendicular height on the plane*



$$FM = \sqrt{\left\{3\cdot5^2 - \left(\frac{5}{2}\right)^2\right\}} = 2\cdot4495 \text{ cm}$$

$$h = FO = \sqrt{\left(2\cdot4495^2\right) - \left(\frac{8-4\cdot5}{2}\right)^2}$$

$$= \sqrt{2\cdot938}$$

$$= 1\cdot714 \text{ cm} \left(4\ s.f\right)$$

(ii) The angle between the face AED and the base ABCD. (4 marks)

Solution

⇒ *AED can be translated onto plane BFC.*

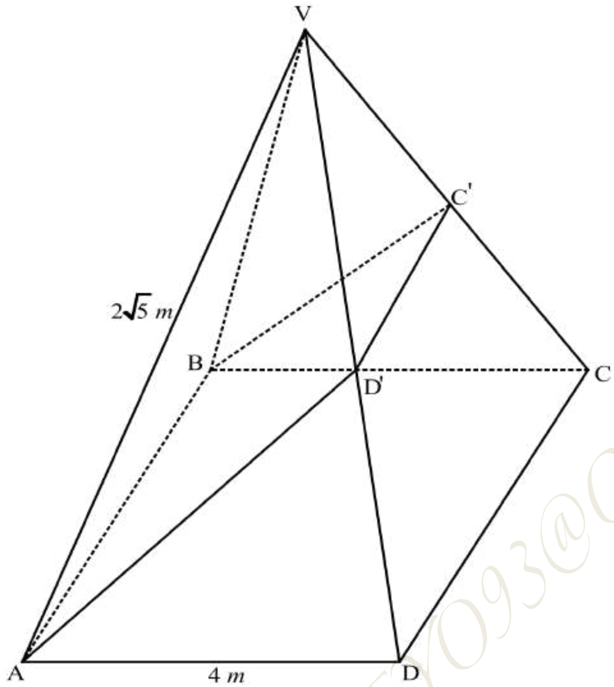
$$\tan\alpha = \frac{1\cdot714}{1\cdot75}$$

$$\alpha = \tan^{-1}\left(\frac{1\cdot714}{1\cdot75}\right)$$

$$= 44\cdot40^0 \left(4\ s.f\right)$$

3. **1992 Paper 2 Number 19**

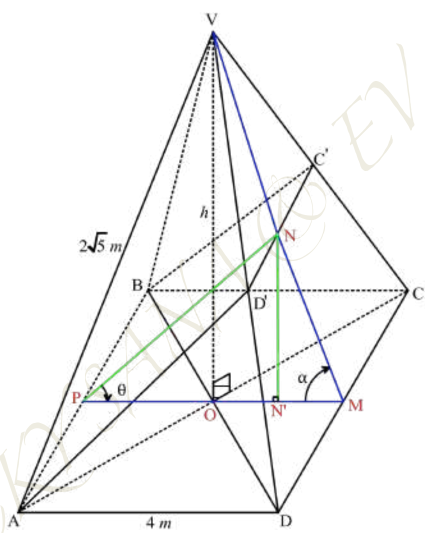
A right pyramid VABCD has a square base ABCD of sides 4 m.The slant edges VA,VB,VC and VD are $2\sqrt{5}$ m long.



(a) Calculate;

(i) the height of the pyramid. (3 marks)

Solution



$$AO = \frac{1}{2}(AC) = \frac{1}{2}\left(\sqrt{4^2+4^2}\right) = 2\sqrt{2}$$

$$h = VO = \sqrt{\left(2\sqrt{5}\right)^2 - \left(2\sqrt{2}\right)^2}$$

$$= \sqrt{12} = 2\sqrt{3}\text{ m}$$

$$= 3\cdot 464\text{ m}$$

(ii) the angle between the plane VAB and the base ABCD. (2 marks)

Solution

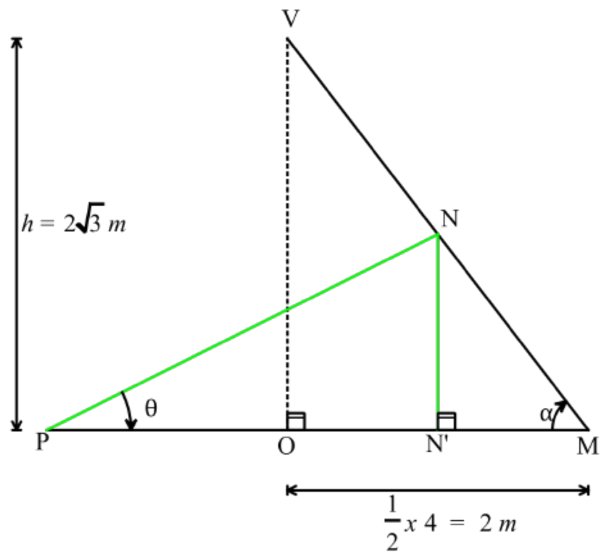
$$\tan\alpha = \frac{\cancel{2}\sqrt{3}}{\cancel{2}}$$

$$\alpha = \tan^{-1}\left(\sqrt{3}\right)$$

$$= 60^0$$

(b) C' and D' are midpoints of VC and VD respectively. Calculate the angle between the planes ABCD and $ABC'D'$ (3 marks)

Solution



$$NN' = \frac{1}{2}VO = \frac{1}{2}\left(2\sqrt{3}\right) = \sqrt{3}\text{ m}$$

Let $N'M = x$ then $ON' = 2 - x$

$$\frac{x}{2} = \frac{\sqrt{3}}{2\sqrt{3}}$$

$$x = \left(\frac{\sqrt{3}}{2\sqrt{3}}\right) \times 2$$

$$= 1\text{ m}$$

$$ON' = 2 - x = 2 - 1 = 1\text{ m}$$

$$PN' = PO + ON' = \frac{1}{2}(4) + 1 = 3\text{ m}$$

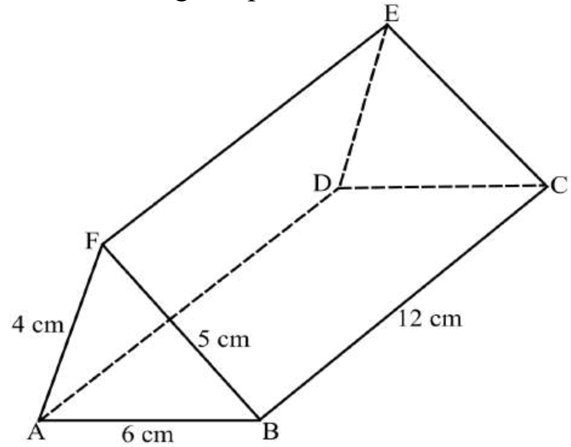
$$\tan\theta = \frac{NN'}{PN'} = \frac{\sqrt{3}}{3}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$$

$$= 30^0$$

4. 1993 paper 2 number 22.

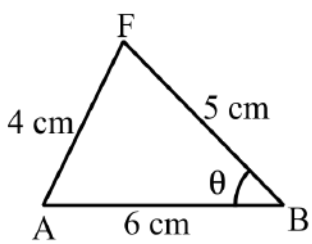
The figure on the next page shows a triangular prism with dimensions as shown.



Calculate:

(a) the angle between the faces FBCE and ABCD. (2 marks)

Solution



➔Since it is a scalene triangle at the face AFB then we use the cosine rule.

$$b^2 = a^2 + f^2 - 2af\cos B$$

$$4^2 = 5^2 + 6^2 - 2(5)(6)\cos B$$

$$\cos B = \frac{(5^2 + 6^2) - 4^2}{2(5)(6)} = \frac{45}{60}$$

$$B = \cos^{-1}\left(\frac{45}{60}\right)$$

$$= 41\cdot 41^0 \,(4\ s.f)$$

(b) the volume of the prism. (3 marks)

Solution

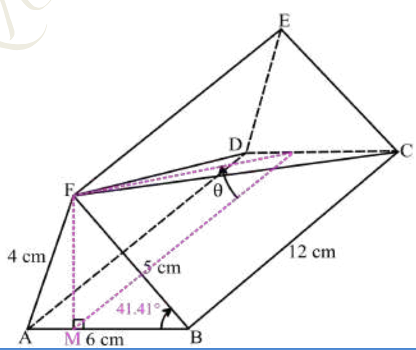
$$\text{Volume} = \left(\frac{1}{2}ab\sin\theta\right)\times l$$

$$= \frac{1}{2}\times 6\times 5(\sin 41\cdot 41^0)\times 12$$

$$= 119\cdot 05\ \text{cm}^3$$

(c) the angle between the planes DFC and ABCD. (3 marks)

Solution



$$FM = 5(\sin 41\cdot 41^0) = 3\cdot 3071\ \text{cm}$$

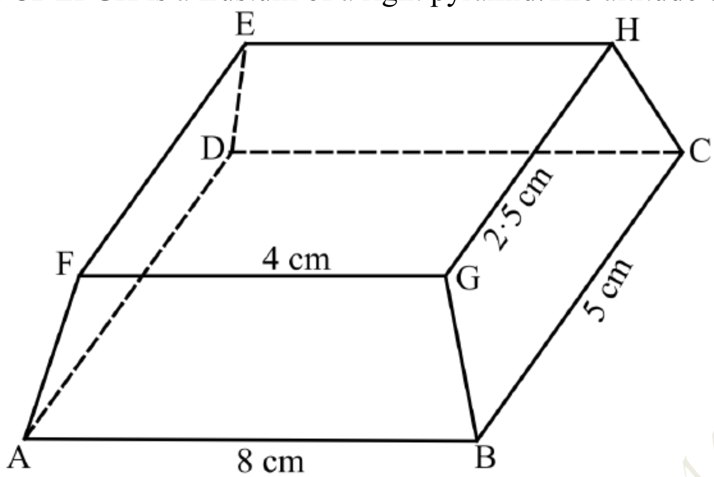
$$\tan\theta = \frac{3\cdot 3071}{12}$$

$$\theta = \tan^{-1}\left(\frac{3\cdot 3071}{12}\right)$$

$$= 15\cdot 41^0\,(4\ s.f)$$

5. **1994 paper 1 number 22**

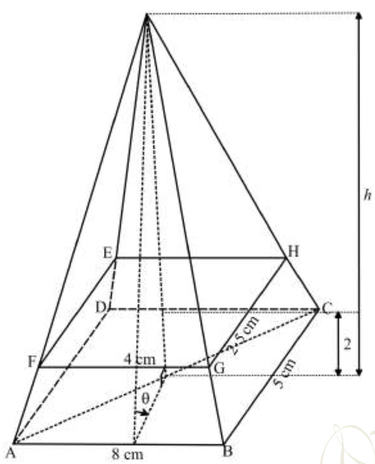
In the figure below ABCDEFGH is a frustum of a right pyramid.The altitude of the frustum is 2 cm.



Calculate:

(a) the altitude of the pyramid. (2 marks)

Solution



By similarity and elargment we have;

$$\frac{4}{8}=\frac{h-2}{h}$$

$$8(h-2)=4h$$

$$8h-4h=16$$

$$h=\frac{16}{4}$$

$$=4\text{ cm}$$

(b) the volume of the frustum. (3 marks)

Solution

$$\text{Volume}=\left(\frac{1}{3}\times 8\times 5\times 4\right)-\left(\frac{1}{3}\times 4\times 2\cdot 5\times 2\right)$$

$$=\frac{160}{3}-\frac{20}{3}=46\frac{2}{3}\text{ cm}^3$$

$$=46\cdot 67\text{ cm}^3$$

(c) the angle between the base of the frustum and the face ABGF. (3 marks)

Solution

$$\tan\theta = \frac{4}{\frac{1}{2}(5)}$$

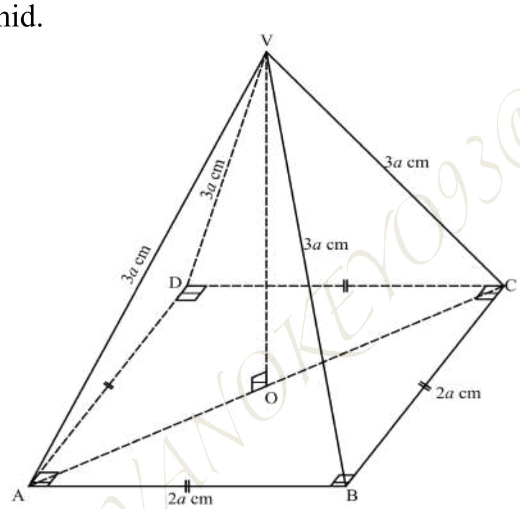
$$\theta = \tan^{-1}\left\{\frac{4}{\frac{1}{2}(5)}\right\} = 57\cdot 99561679^{0}$$

$$= 58^{0}$$

6. 1996 paper 2 number 13

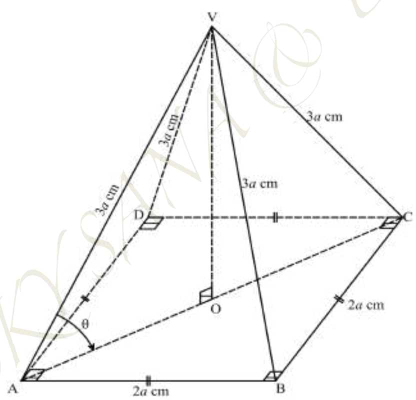
The base of a right pyramid is a square ABCD of side $2a$ cm .The slant edges VA,VB,VC and VD are each of length $3a$ cm .

(a) Sketch and label the pyramid. (1 mark)



(b) Find the angle between a slanting edge and the base. (2 marks)

Solution



$$AC = \sqrt{(2a)^2 + (2a)^2} = 2a\sqrt{2} \text{ cm}$$

$$AO = \frac{1}{2} \times AC = \frac{1}{2}\left(2a\sqrt{2}\right) = a\sqrt{2} \text{ cm}$$

$$\cos\theta = \frac{AO}{AV} = \frac{a\sqrt{2}}{3a} = \frac{\sqrt{2}}{3}$$

$$= \sin^{-1}\left(\frac{\sqrt{2}}{3}\right)$$

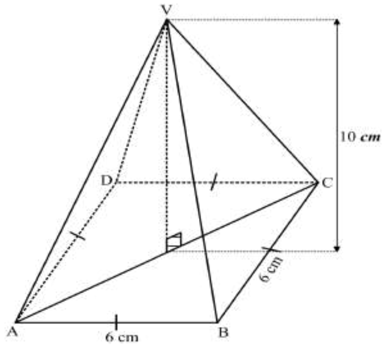
$$= 61\cdot 87^{0}$$

7. 1997 paper 2 number 6

A pyramid of height 10 cm stands on a square base ABCD of side 6 cm.

(a) Draw a sketch of the pyramid. (1 mark)

Solution



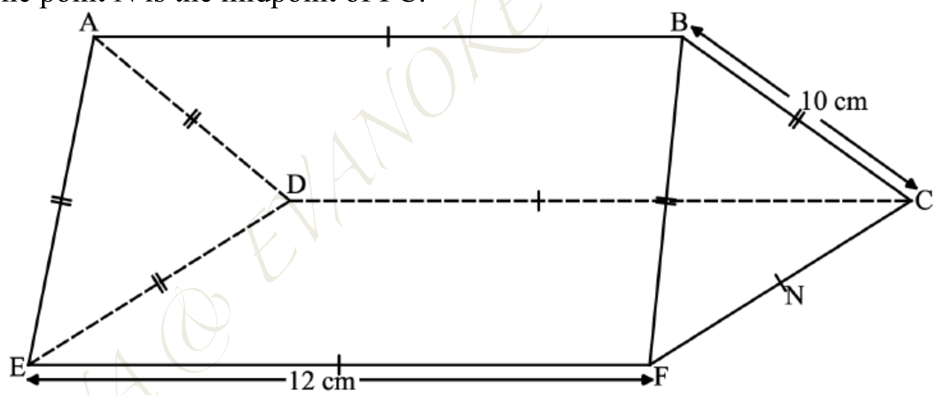
(b) Calculate the perpendicular distance from the vertex to the side AB. (2 marks)

Solution

$$\perp \text{ distance from V to AB} = \sqrt{10^2 + \left(\frac{1}{2}\times 6\right)^2} = \sqrt{109} = 10\cdot 44 \text{ cm}$$

8. 1998 paper 2 number 16

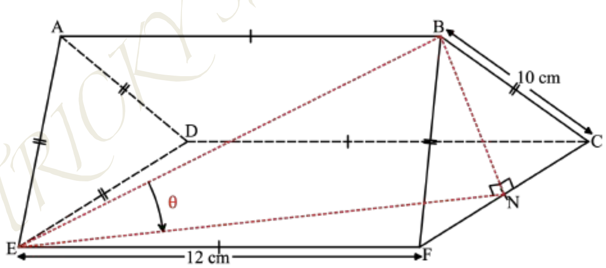
The triangular prism shown below has the sides AB = DC = EF = 12 cm. The ends are equilateral triangles of sides 10 cm. The point N is the midpoint of FC.



(a) Find the length of:

(i) BN (1 mark)

Solution



$$BN = \sqrt{(10^2 - 5^2)} = \sqrt{75} = 5\sqrt{3} = 8\cdot 660 \text{ cm}$$

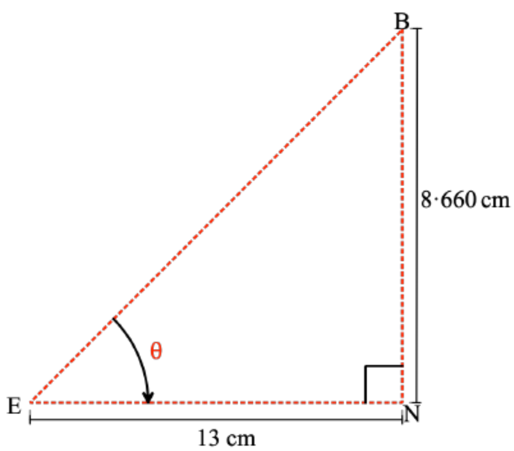
(ii) EN (1 mark)

Solution

$$EN = \sqrt{(12^2 + 5^2)} = \sqrt{169} = 13 \text{ cm}$$

(b) Find the angle between the line EB and the plane CDEF. (2 marks)

Solution



$$\tan\theta = \frac{NB}{NE} = \frac{8\cdot660}{13}$$

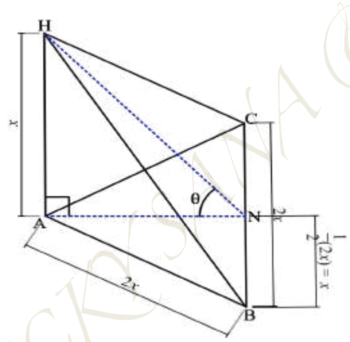
$$\theta = \tan^{-1}\left(\frac{8\cdot660}{13}\right)$$

$$= 33\cdot67^0 \,(4\ s.f)$$

9. **1999 paper 1 number 14**

An equilateral triangle ABC lies in a horizontal plane. A vertical flag AH stands at A.If $AB = 2AH$,find the angle between the planes ABC and HBC. (3 marks)

Solution



$$AN = \sqrt{\{(2x)^2 - x^2\}} = \sqrt{3x^2} = x\sqrt{3}\text{units}$$

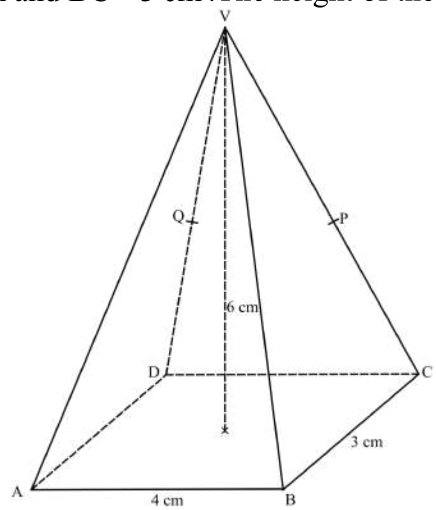
$$\tan\theta = \frac{x}{x\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= 30^0$$

10. 1999 paper 2 number 24

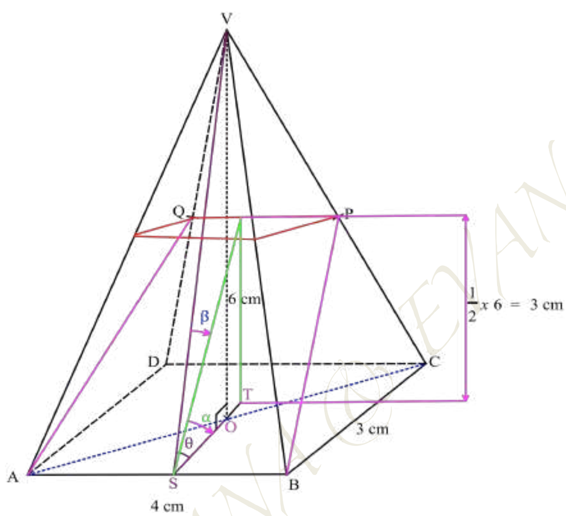
The diagram below shows a right pyramid VABCD with V as the vertex. The base of the pyramid is a rectangle ABCD with AB = 4 cm and BC = 3 cm .The height of the pyramid is 6 cm.



(a) Calculate the;

(i) length of the projection of VA on the base. (2 marks)

Solution



$$AO = \frac{1}{2}AC = \frac{1}{2}\left(\sqrt{4^2+3^2}\right) = \frac{1}{2}\left(\sqrt{25}\right) = 2\cdot5 \text{ cm}$$

(ii) angle between the face VAB and the base. (2 marks)

Solution

It is the angle marked θ on thediagram in (a) (i) above.

$$\tan\theta = \frac{VO}{SO} = \frac{6}{\frac{1}{2}(3)} = \frac{6}{1\cdot5} = 4$$

$$\theta = \tan^{-1}(4)$$

$$= 75\cdot96(4\ s.f)$$

(b) P is the midpoint of VC and Q is the midpoint of VD.Find the angle between the planes VAB and the plane ABPQ. (4 marks)

Solution

$$OT = \frac{1}{2}\left(\frac{1}{2}\times 3\right) = 0\cdot 75 \text{ cm}$$

$$\tan\alpha = \frac{3}{ST} = \frac{3}{SO+OT} = \frac{3}{\left(\frac{1}{2}\times 3\right)+\left(\frac{1}{2}\left(\frac{1}{2}\times 3\right)\right)} = \frac{3}{2\cdot 25}$$

$$\alpha = \tan^{-1}\left(\frac{3}{2\cdot 25}\right)$$

$$= 53\cdot 13^0$$

The required angle is β as marked on the diagram in (a) (i) above.

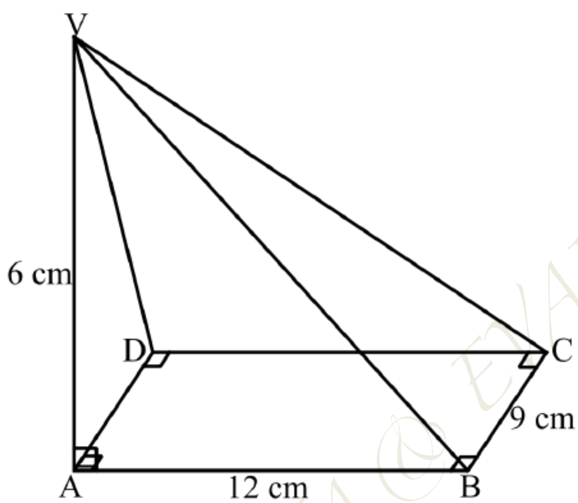
$$\beta = 75\cdot 96^0 - 53\cdot 13^0$$

$$= 22\cdot 83^0$$

11. 2000 paper 1 number 11.

A pyramid VABCD has a rectangular horizontal base ABCD with AB = 12 cm and BC = 9 cm.The vertex V is vertically above A and VA = 6 cm. Calculate the volume of the pyramid. (2 marks)

Solution



$$\text{Volume} = \frac{1}{3}\times \text{Base area}\times \text{height}$$

$$= \frac{1}{3}\times 12\times 9\times 6$$

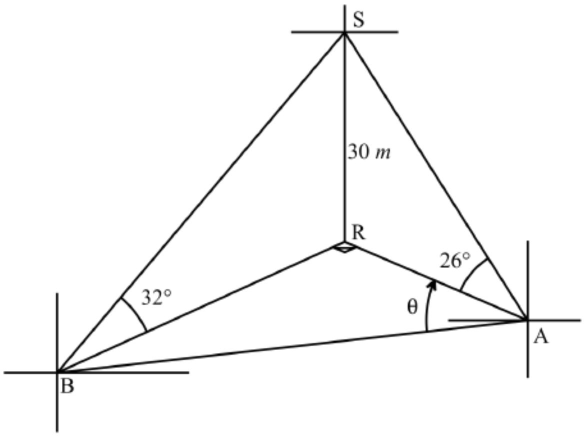
$$= 216 \text{ cm}^3$$

12. **2001 paper 2 number 20.**

An electric pylon is 30 m high. A point S on top of the pylon is vertically above another point R on the ground. Points A and B are on the same horizontal ground as R.Point A is due south of the pylon and the angle of elevation of S from A is 26º.Point B is due west of the pylon and the angle of elevation of S from B is 32º.Calculate:

(a) distance from A to B. (6 marks)

Solution



$$BR = \frac{30}{\left(\tan 32^0\right)} = 48 \cdot 01 \text{ and } AR = \frac{30}{\left(\tan 26^0\right)} = 61 \cdot 51$$

$$AB = \sqrt{(48 \cdot 01)^2 + (61 \cdot 51)^2}$$

$$= \sqrt{6{,}088 \cdot 4402}$$

$$= 78 \cdot 028 \text{ m}$$

(b) bearing of B from A. (2 marks)

Solution

$$\tan\theta = \frac{BR}{AR} = \frac{48 \cdot 01}{61 \cdot 51}$$

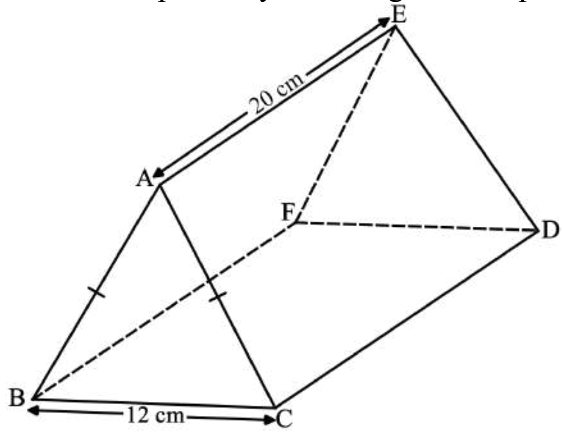
$$\theta = \tan^{-1}\left(\frac{48 \cdot 01}{61 \cdot 51}\right)$$

$$= 37 \cdot 97^0$$

$$= N37 \cdot 97^0 W \text{ or } \left(360^\circ - 37 \cdot 97\right)^0 = 322 \cdot 03^0$$

13. **2002 paper 1 number 18.**

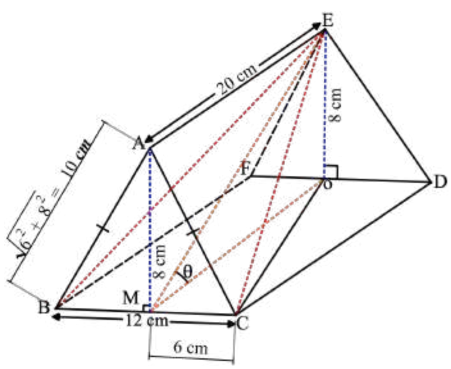
The figure below represents a right prism whose triangular faces are isosceles. The base and height of each triangular face are 12 cm and 8 cm respectively. The length of the prism is 20 cm.



Calculate the:

(a) length CE. (3 marks)

Solution



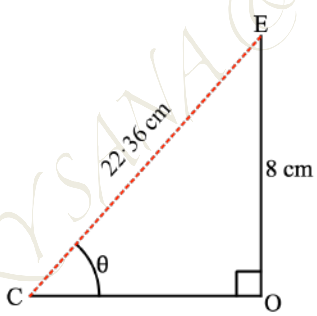
$$\left(\sqrt{6^2+8^2}\right)=\sqrt{100}=10 \text{ cm}$$

$$CE=\sqrt{20^2+10^2}=\sqrt{500}=22\cdot 36\,(4\ s.f)$$

(b) angle between

(i) The line CE and the plane BCDF. (3 marks)

Solution



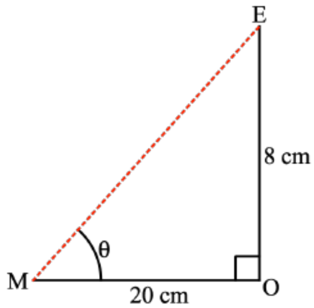
$$\sin\theta=\frac{8}{22\cdot 36}$$

$$\theta=\sin^{-1}\left(\frac{8}{22\cdot 36}\right)$$

$$=20\cdot 97^0\,(4\ s.f)$$

(ii) The plane EBC and the base BCDF. (2 marks)

Solution



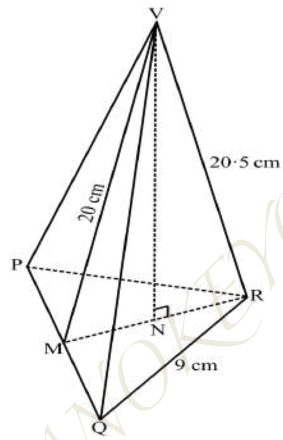
$$\tan\theta = \frac{8}{20}$$

$$\theta = \tan^{-1}\left(\frac{8}{20}\right)$$

$$= 21\cdot 80^0\,(4\ s.f)$$

14. 2002 paper 2 number 20.

The figure below VPQR below represents a model of a top of a tower. The horizontal base PQR is an equilateral triangle of side 9 cm.The lengths of the edges are VP = VQ = VR=20·5 cm .Point M is the mid point of PQ and VM = 20 cm .Point N is on the base and vertically below V.



Calculate the:

(a) (i) length of RM. (2 marks)

Solution

$$RM = \sqrt{\left\{9^2 - \left(\frac{1}{2}\times 9\right)^2\right\}}$$

$$= \sqrt{60\cdot 75}$$

$$= 7\cdot 794\ \text{cm}\,(4\ s.f)$$

(ii) Height of the model. (2 marks)

Solution

$$RN = \frac{2}{3}RM = \frac{2}{3}\times 7\cdot 794 = 5\cdot 196\ \text{cm}$$

$$VN = \sqrt{\left(20\cdot 5^2 - 5\cdot 196^2\right)} = \sqrt{393\cdot 25}$$

$$= 19\cdot 83\,(4\ s.f)$$

(iii) Volume of the model. (2 marks)

Solution

$$\text{Volume} = \frac{1}{3}\text{Base area} \times \text{height}$$

$$\text{VN} = \frac{1}{3}\left(\frac{1}{2} \times 9 \times 7 \cdot 749\right) \times 19 \cdot 83$$

$$= 231 \cdot 83 \text{ cm}^3$$

(b) The model is made of material whose density is $2{,}700 \text{ kg/m}^3$. Find the mass of the model. (2 marks)

Solution

$$\text{Mass} = \text{Density} \times \text{volume}$$

$$= \frac{2{,}700}{1{,}000} \times 231 \cdot 83$$

$$= 625 \cdot 948 \text{ g}$$

$$\text{or Mass} = \text{Density} \times \text{volume}$$

$$= \frac{2{,}700}{1{,}000} \times 231 \cdot 83$$

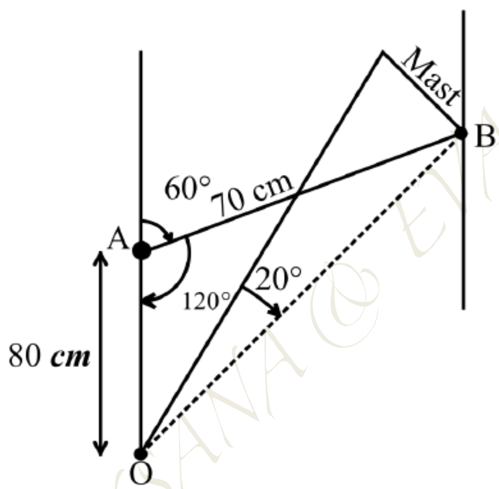
$$= 625 \cdot 948 \text{ g} \div 1000\text{kg} = 0 \cdot 62595 \text{ kg}$$

15. 2003 paper 1 number 15

The points O,A and B are on the same horizontal ground. Point A is 80 metres to north of O.Point B is located 70 metres on a bearing of 060^0 from A. A vertical mast stands at point B.The angle of elevation of the top of the mass from O is 20^0. Calculate:

(a) The distance of B from O. (2 marks)

Solution



By cosine rule we have;

$$\text{OB}^2 = 80^2 + 70^2 - 2 \times 80 \times 70\cos 120^0$$

$$\text{OB} = \sqrt{\left\{\left(80^2 + 70^2\right) - \left(2 \times 80 \times 70\cos 120^0\right)\right\}}$$

$$= \sqrt{16{,}900}$$

$$= 130 \text{ cm}$$

(b) The height of the mast in metres. (2 marks)

Solution

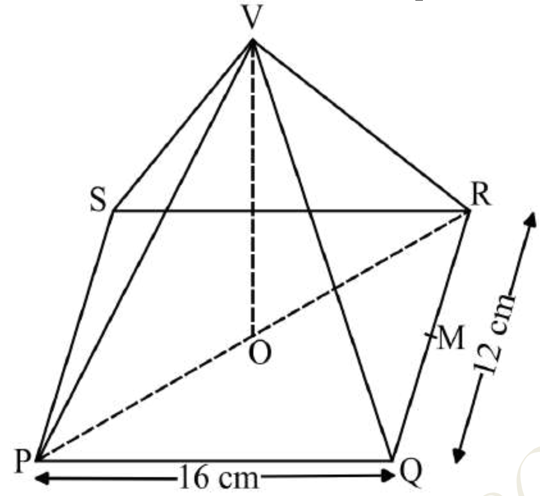
$$\tan 20^0 = \frac{h}{130}$$

$$h = 130 \times \tan 20^0$$

$$= 47 \cdot 32\ m$$

16. 2003 paper 2 number 24

The figure below represents a right pyramid with vertex V and a rectangular base PQRS.
VP = VQ = VR = VS = 18 cm.PQ = 16 cm.M and O are the midpoints of QR and PR respectively.



Find:

(a) the length of the projection of line VP on the plane PQRS. (2 marks)

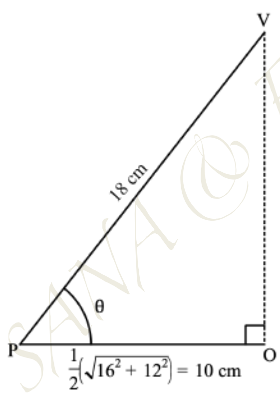
Solution

Line PO is the projection of line VP on the base PQRS.

$$PO = \frac{1}{2}\left\{\sqrt{\left(16^2 + 12^2\right)}\right\} = \frac{1}{2}\{20\} = 10 \text{ cm}$$

(b) the size of the angle between the line VP and the plane PQRS. (2 marks)

Solution



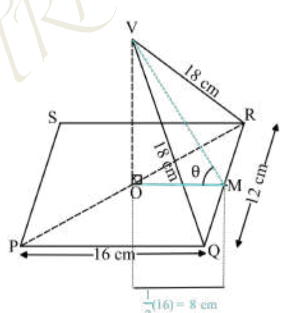
$$\cos\theta = \frac{10}{18}$$

$$\theta = \cos^{-1}\left(\frac{10}{18}\right)$$

$$= 56\cdot25^0\ (4\ s.f)$$

(c) the size of the angle between the planes VDR and PQRS. (4 marks)

Solution



$$VO = \sqrt{\left(18^2 - 10^2\right)} = 4\sqrt{14} \text{ cm}$$

$$\tan\theta = \frac{4\sqrt{14}}{8}$$

$$\theta = \tan^{-1}\left(\frac{4\sqrt{14}}{8}\right)$$

$$= 61\cdot87^0$$

Alternatively;

$$VM = \sqrt{\left(18^2 - 6^2\right)} = 16\cdot97 \text{ cm}$$

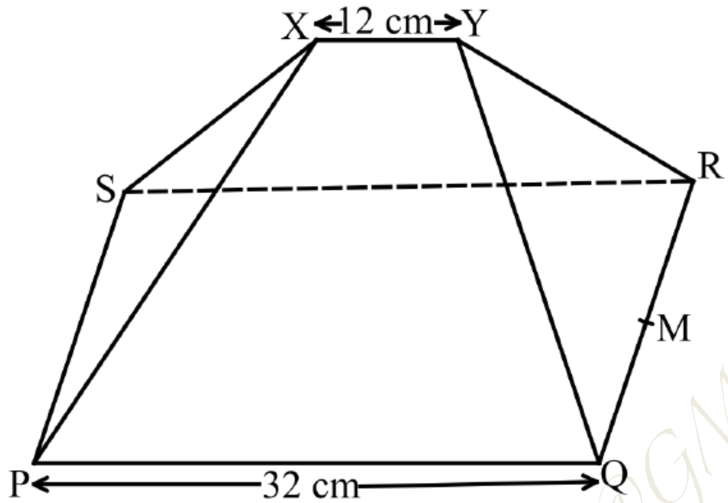
$$\cos\ \theta = \frac{8}{16\cdot97}$$

$$\theta = \cos^{-1}\left(\frac{8}{16\cdot97}\right)$$

$$= 61\cdot87^0\ (4\ s.f)$$

17. 2004 paper 2 number 24

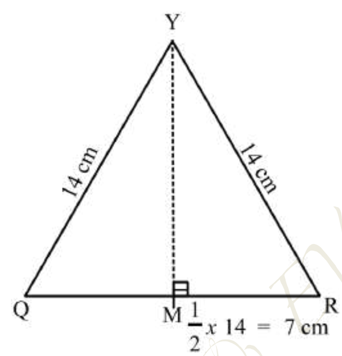
The figure below shows a model of a roof with a rectangular base PQRS.PQ = 32 cm and QR = 14 cm. The ridge XY = 12 cm and is centrally placed.The faces PSX and QRY are equilateral triangles.M is the midpoint of QR.



Calculate:

(a) (i) the length of YM. (1 mark)

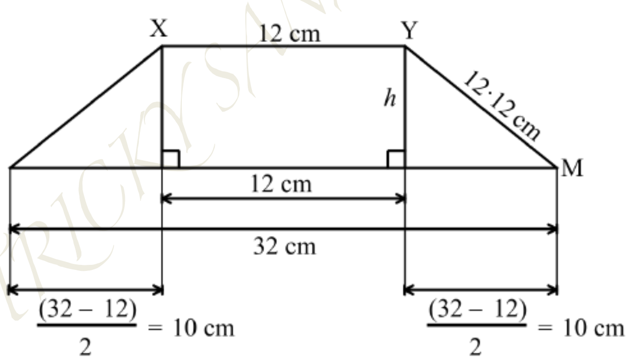
Solution



$$YM = \sqrt{14^2 - 7^2} = \sqrt{147} = 12 \cdot 12 \text{ cm}$$

(ii) the height of Y above the base PQRS. (2 marks)

Solution



$$h = \sqrt{\left(12 \cdot 12^2 - 10^2\right)} = \sqrt{4 \cdot 8944} = 6 \cdot 848 \text{ cm}\left(4\ s.f\right)$$

(b) the angle between the planes RSXY and PQRS. (3 marks)

Solution

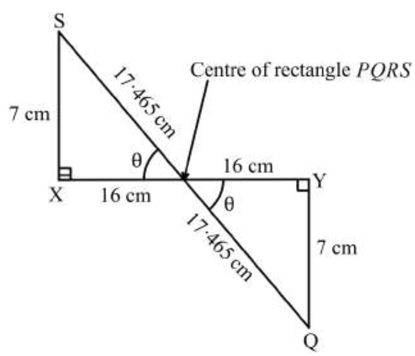
$$\tan\theta = \frac{h}{MR} = \frac{6\cdot 878}{\frac{1}{2}(14)}$$

$$\theta = \tan^{-1}\left(\frac{6\cdot 878}{7}\right)$$

$$= 44\cdot 37^0 \left(4\ s.f\right)$$

(c) the acute angle between the lines XY and QS. (2 marks)

Solution



$$SQ = \sqrt{(32^2 + 14^2)} = \sqrt{1220} = 34\cdot 93 \text{ cm}$$

$$\tan\theta = \frac{7}{16}$$

$$\theta = \tan^{-1}\left(\frac{7}{16}\right)$$

$$= 23\cdot 63^0 \left(4\ s.f\right)$$

$$SQ = \sqrt{(32^2 + 14^2)} = \sqrt{1220} = 34\cdot 93 \text{ cm}$$

$$\cos\theta = \frac{16}{17\cdot 465}$$

$$\theta = \cos^{-1}\left(\frac{16}{17\cdot 47}\right)$$

$$= 23\cdot 635^0$$

$$SQ = \sqrt{(32^2 + 14^2)} = \sqrt{1220} = 34\cdot 93 \text{ cm}$$

$$\sin\theta = \frac{7}{17\cdot 465}$$

$$\theta = \sin^{-1}\left(\frac{7}{17\cdot 465}\right)$$

$$= 23\cdot 63^0 \left(4\ s.f\right)$$

By cosine rule;

$$7^2 = 17\cdot 465^2 + 16^2 - 2\left(16 \times 17\cdot 465\right)\cos\theta$$

$$\cos\theta = \frac{7^2 - \left(17\cdot 465^2 + 16^2\right)}{-2\left(16 \times 17\cdot 465\right)}$$

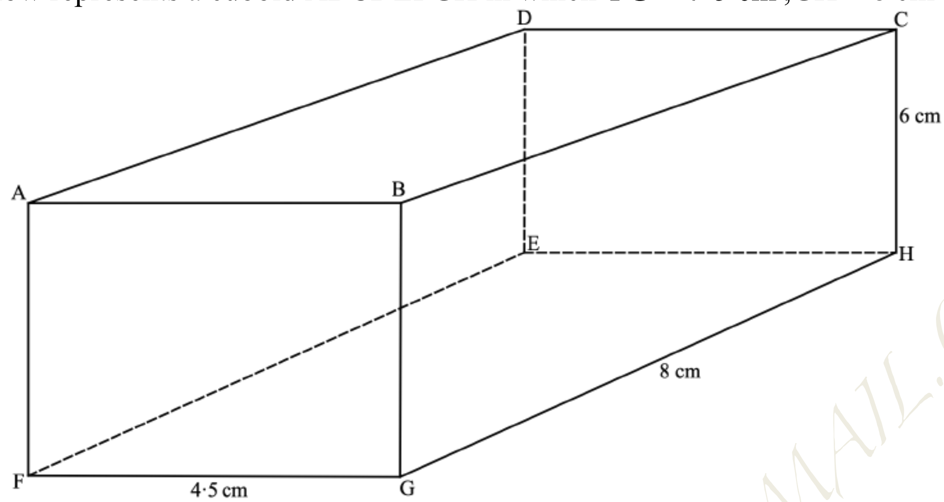
$$\theta = \cos^{-1}\left\{\frac{7^2 - \left(17\cdot 465^2 + 16^2\right)}{-2\left(16 \times 17\cdot 465\right)}\right\}$$

$$= 23\cdot 63^0 \left(4\ s.f\right)$$

Accept any method.

18. 2005 paper 2 number 23

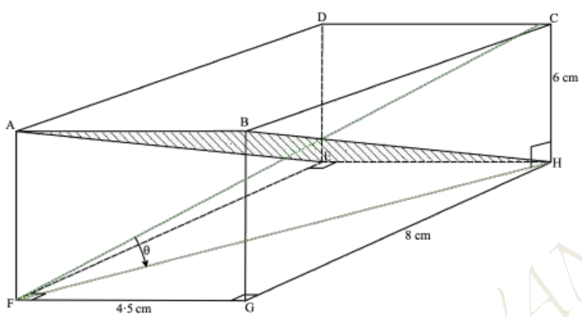
The diagram below represents a cuboid ABCDEFGH in which $FG = 4\cdot5$ cm, $GH = 8$ cm and $HC = 6$ cm.



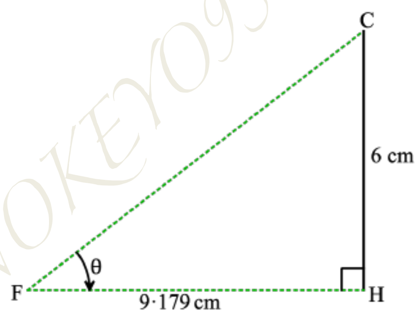
Calculate:

(a) the length of FC. (2 marks)

Solution



$$FG = \sqrt{\left(4\cdot5^2 + 8^2\right)} = 9\cdot179 \text{ cm}$$



$$FC = \sqrt{\left(9\cdot179\right)^2 + 6^2} = \sqrt{120\cdot25} = 10\cdot97 \text{ cm}\left(4\ s.f\right)$$

(b) (i) the size of the angle between the lines FC and FH. (2 marks)

Solution

$$\tan\theta = \frac{HC}{HF} = \frac{6}{9\cdot179}$$

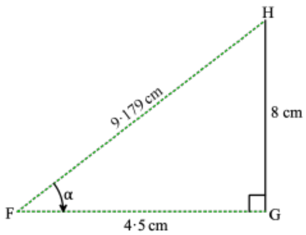
$$\theta = \tan^{-1}\left(\frac{6}{9\cdot179}\right)$$

$$= 33\cdot17\left(4s\cdot f\right)$$

(ii) the size of the angle between the lines AB and FH. (2 marks)

Solution

Translate line AB onto FG and joins FH where F is the common angle required.



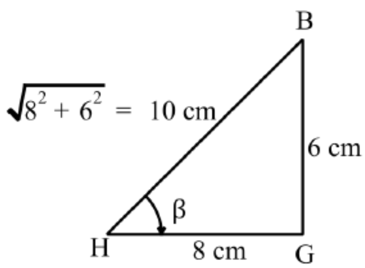
$$\tan\alpha = \frac{HG}{GF} = \frac{8}{4\cdot5}$$
$$\alpha = \tan^{-1}\left(\frac{8}{4\cdot5}\right)$$
$$= 60\cdot64^0\,(4\ s.f)$$

$$\cos\alpha = \frac{4\cdot5}{9\cdot179}$$
$$\alpha = \cos^{-1}\left(\frac{4\cdot5}{9\cdot179}\right)$$
$$= 60\cdot64^0\,(4\ s.f)$$

$$\sin\alpha = \frac{8}{9\cdot179}$$
$$\alpha = \tan^{-1}\left(\frac{8}{9\cdot179}\right)$$
$$= 60\cdot64^0\,(4\ s.f)$$

(c) the size of the angle between the plane ABHE and the plane FGHE. (2 marks)

Solution



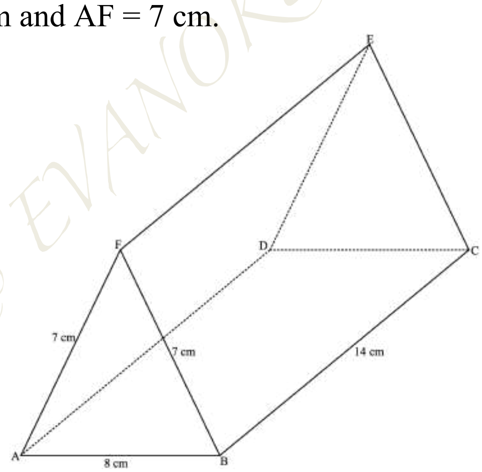
$$\tan\beta = \frac{6}{8}$$
$$\beta = \tan^{-1}\left(\frac{6}{8}\right)$$
$$= 36\cdot87^0\,(4\ s.f)$$

$$\cos\beta = \frac{8}{10}$$
$$\beta = \cos^{-1}\left(\frac{8}{10}\right)$$
$$= 36\cdot87^0\,(4\ s.f)$$

$$\sin\beta = \frac{6}{10}$$
$$\alpha = \sin^{-1}\left(\frac{6}{10}\right)$$
$$= 36\cdot87^0\,(4\ s.f)$$

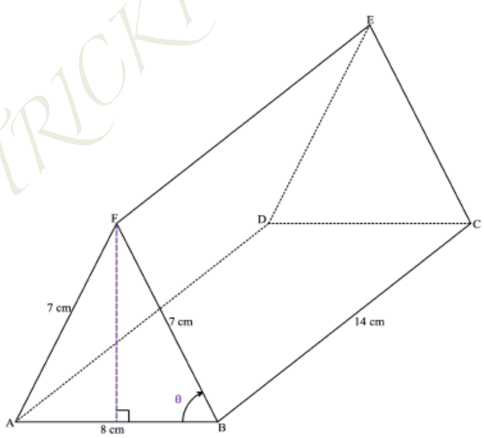
19. 2008 paper 2 number 14

The figure below represents a triangular prism. The faces ABCD,ADEF and CBFE are rectangles. AB = 8 cm,BC = 14 cm,BF = 7 cm and AF = 7 cm.



Calculate the angle between faces BCEF and ABCD. (3 marks)

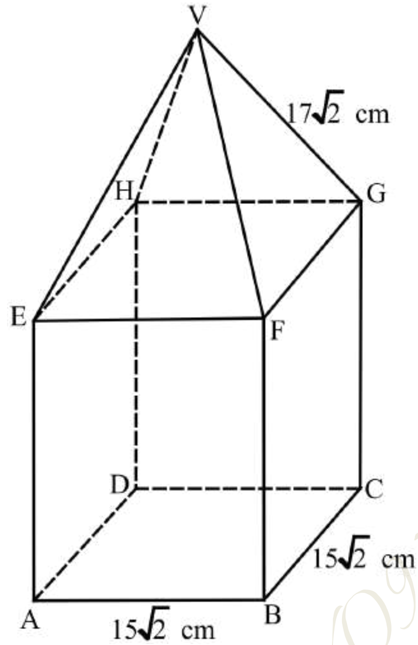
Solution



$$\cos\theta = \frac{\frac{1}{2}(8)}{7}$$
$$\theta = \cos^{-1}\left(\frac{\frac{1}{2}(8)}{7}\right)$$
$$= 55\cdot15^0\,(4\ s.f)$$

20. 2009 paper 2 number 22

The figure below shows a right pyramid mounted onto a cuboid. $AB = BC = 15\sqrt{2}$ cm. $CG = 8$ cm and $VG = 17\sqrt{2}$ cm



Calculate:

(a) the length AC; (1 mark)

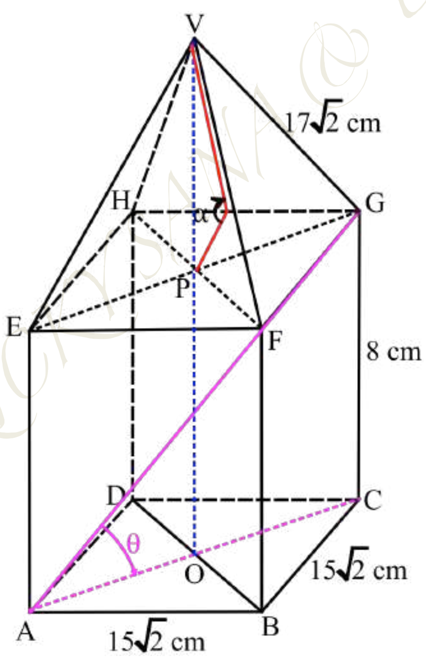
Solution

$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{\left(15\sqrt{2}\right)^2 + \left(15\sqrt{2}\right)^2} = \sqrt{900} = 30 \text{ m}$$

(b) the angle between the line AG and the plane ABCD; (3 marks)

Solution



$$\tan\theta = \frac{CG}{AC} = \frac{8}{30} \text{ or equivalent}$$

$$\theta = \tan^{-1}\left(\frac{8}{30}\right)$$

$$= 14 \cdot 93^0$$

(c) the vertical height of point V from the plane ABCD; (3 marks)

Solution

$$\text{Pyramid height, } VP = \sqrt{VG^2 - PG^2}; VG = 17\sqrt{2}, PG = \frac{1}{2} \times 30 = 15 \text{ cm}$$

$$= \sqrt{\left(17\sqrt{2}\right)^2 - 15^2} = 18 \cdot 79 \text{ cm}$$

$$VO = OP + VP$$

$$= 8 + 18 \cdot 79$$

$$= 26 \cdot 79 \text{ cm}$$

(d) the angle between the planes EFV and ABCD. (3 marks)

Solution

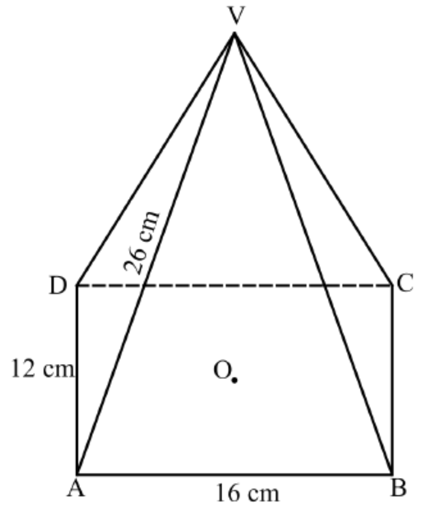
$$\tan\alpha = \frac{VP}{\frac{1}{2} \times FG} = \frac{18 \cdot 79}{\frac{1}{2} \times 15\sqrt{2}}$$

$$\theta = \tan^{-1}\left(\frac{18 \cdot 79}{7 \cdot 5\sqrt{2}}\right)$$

$$= 60 \cdot 56^0$$

21. 2011 paper 2 number 22

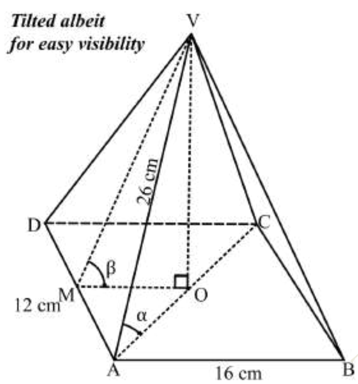
The figure below represents a rectangular based pyramid VABCD.AB = 12 cm and AD = 16 cm.Point O is vertically below V and VA = 26 cm.



Calculate:

(a) the height,VO,of the pyramid; (4 marks)

Solution



$$AC = 16^2 + 12^2 = 400$$

$$AC = \sqrt{400} = 20 \text{ cm}$$

$$AO = \frac{1}{2} \times 20 \text{ cm} = 10 \text{ cm}$$

$$VO = \sqrt{26^2 - 10^2} = 24 \text{ cm}$$

(b) the angle between the edge VA and the plane ABCD; (3 marks)

Solution

$$\tan\alpha = \frac{VO}{AO} = \frac{24}{10} = 2\cdot4$$

$$\alpha = \tan^{-1}(2\cdot4)$$

$$= 67\cdot38^0$$

(c) the angle between the planes VAB and ABCD. (3 marks)

Solution

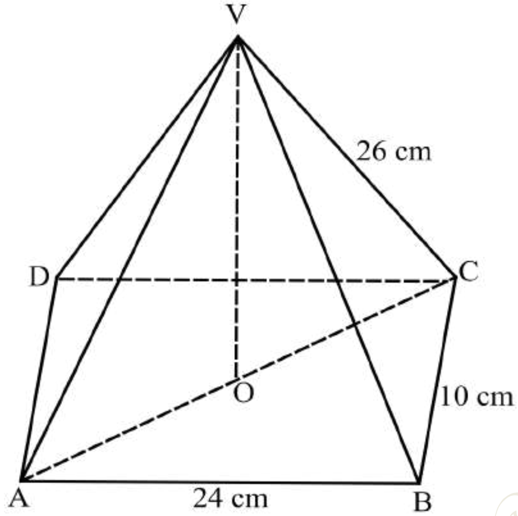
$$\tan\beta = \frac{VO}{MO} = \frac{24}{\frac{1}{2}(16)} = 3$$

$$\alpha = \tan^{-1}(3)$$

$$= 71\cdot57^0$$

22. 2012 paper 2 number 16

In the figure below, VABCD is a right pyramid on a rectangular base. Point O is vertically below the vertex V. AB = 24 cm, BC = 10 cm and CV = 26 cm.



Calculate the angle between the edge CV and the base ABCD. (3 marks)

Solution

$$OC = \frac{1}{2}\left(\sqrt{24^2 + 10^2}\right) \qquad M_1$$

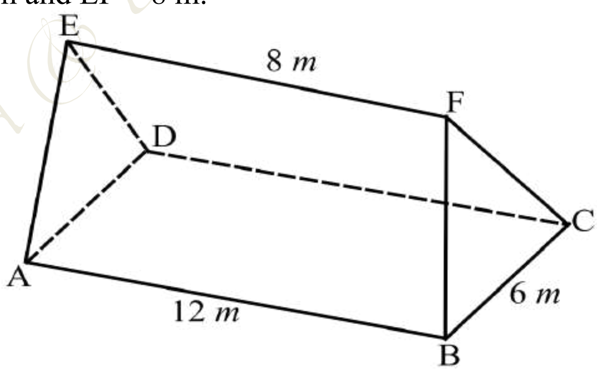
$$= 13 \text{ cm}$$

$$\angle VCO = \cos^{-1}\left(\frac{13}{26}\right) \qquad M_1$$

$$= 60^0 \qquad A_1$$

23. 2013 paper 2 number 20

The figure ABCDEF below represents a roof of a house. AB = DC = 12 m, BC = AD = 6 m, AE = BF = CF = DE = 5 cm and EF = 8 m.



(c) Calculate correct to 2 decimal places, the perpendicular distance of EF from the plane ABCD. (3 marks)

Solution

$$\text{Slant height from F to BC} = \sqrt{5^2 - \left(\frac{1}{2}\times 6\right)^2} = 4 \text{ m}$$

$$\text{Hence, distance of EF from plane ABCD} = \sqrt{4^2 - 2^2} \qquad M_1$$

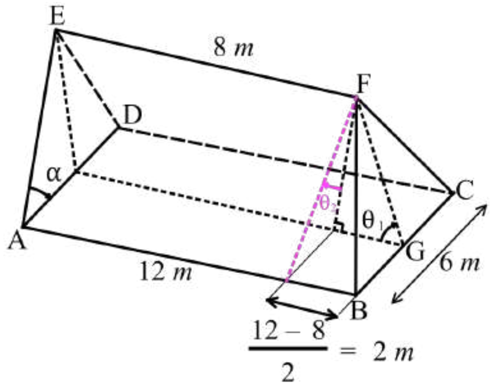
$$= \sqrt{12}$$

$$= 3\cdot 46 \text{ m} \qquad A_1$$

(b) Calculate the angle between:

(i) the planes ADE and ABCD; (2 marks)

Solution



$$\tan\theta_1 = \frac{\perp \text{ distance of EF}}{2} = \frac{\sqrt{12}}{2} \qquad M_1$$

$$\theta_1 = \tan^{-1}\left(\frac{\sqrt{12}}{2}\right)$$

$$= 60^0 \qquad A_1$$

(ii) the line AE and the plane ABCD,correct to 1 decimal place; (2 marks)

Solution

$$\sin\alpha = \frac{\perp \text{ distance of EF}}{AE = FB} = \frac{\sqrt{12}}{5} \text{ or equvalent trig. ratio} \qquad M_1$$

$$\alpha = \sin^{-1}\left(\frac{\sqrt{12}}{5}\right)$$

$$= 43\cdot 9^0 \qquad A_1$$

(iii) the planes ABFE and DCFE,correct to 1 decimal place. (3 marks)

Solution

$$\tan\theta_2 = \frac{\frac{1}{2}\times BC}{\perp \text{ distance of EF}} = \frac{3}{\sqrt{12}} \text{ or equvalent trig. ratio} \qquad M_1$$

$$\theta_2 = \tan^{-1}\left(\frac{3}{\sqrt{12}}\right)$$

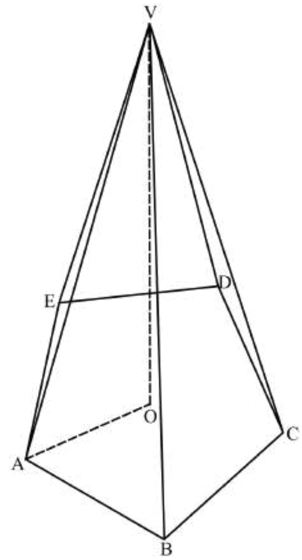
$$= 40\cdot 9^0$$

Angle between planeABFE and DCFE $= 2\times 40\cdot 9^0$ M_1 for doubling

$$= 81\cdot 8^0 \qquad A_1$$

24. **2014 paper 1 number 20**

The figure below shows a right pyramid VABCDE.The base ABCDE is regular pentagon.AO = 15 cm and VO = 36 cm.



Calculate:

(a) the area of the base correct to 2 decimal places; (3 marks)

Solution

$$\text{Base area} = \frac{1}{2}\times 15\times 15\sin\left(\frac{360^0}{5}\right)\times 5 \qquad B_1\ M_1 \quad \text{use of } \sin 72^0$$

$$= 534\cdot 97\ \text{cm}^2 \qquad A_1$$

(b) the length AV; (1 mark)

Solution

$$\text{Length AV} = \sqrt{36^2+15^2} = \sqrt{1521} = 39\ \text{cm} \qquad B_1$$

(c) the surface are of the pyramid correct to 2 decimal places; (4 marks)

Solution

Area of triangular faces;

$$\frac{AB}{\sin\ 72^0} = \frac{15}{\sin\ 54^0}$$

$$AB = \frac{15\sin\ 72^0}{\sin\ 54^0} \qquad M_1$$

$$= 17\cdot 63$$

$$\text{Area} = \sqrt{\left\{\frac{1}{2}(39+39+17\cdot 63)(30\cdot 185)(8\cdot 815^2)\right\}} \qquad M_1 \text{ application of Heron's formula}$$

$$= 334\cdot 89\ \text{cm}^2$$

$$\text{Total area} = (334\cdot 89\times 5)+534\cdot 89 \qquad M_1$$

$$= 2{,}209\cdot 42 \qquad A_1$$

(d) the volume of the pyramid correct to 4 significant figures. (2 marks)

Solution

$$\text{Volume of pyramid} = \text{ Base area} \times \text{height}$$

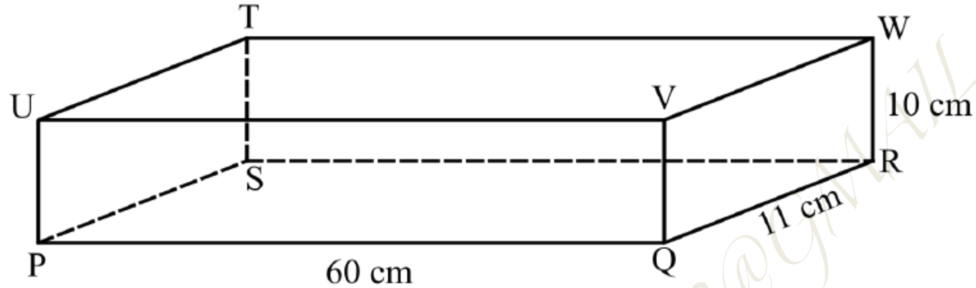
$$= \frac{1}{3} \times 534 \cdot 97 \times 36 \qquad M_1$$

$$= 6{,}419 \cdot 63 \text{ cm}^3$$

$$\simeq 6{,}420 (4\ s.f) \qquad A_1$$

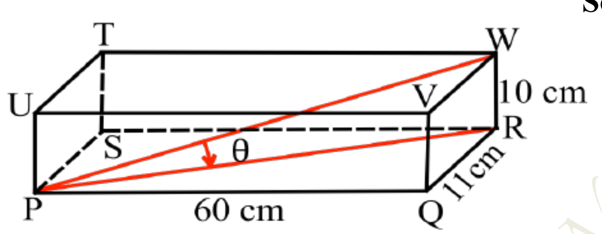
25. 2014 paper 2 number 10

The figure below represents a cuboid PQRSTUVW.PQ = 60 cm,QR = 11 cm and RW = 10 cm.



Calculate the angle between the line PW and plane PQRS, correct to 2 decimal places. (3 marks)

Solution



$$PR = \sqrt{60^2 + 11^2} = 61 \text{ cm} \qquad B_1$$

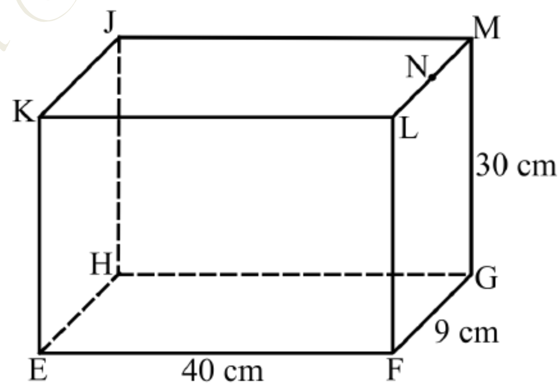
$$\tan\ \theta = \frac{10}{61}$$

$$\theta = \tan^{-1}\left(\frac{10}{61}\right) \qquad M_1$$

$$= 9 \cdot 31^0 \qquad A_1$$

26. 2015 paper 2 number 20

The figure below represents a cuboid EFGHJKLM in which EF = 40 cm,FG = 9 cm,GM = 30 cm.N is the midpoint of LM.



Calculate correct to 4 significant figures:

(a) the length of GL; (1 mark)

Solution

$$GL = \sqrt{30^2 + 9^2} = 31 \cdot 32\text{cm} \qquad B_1$$

(b) the length of FJ: (2 marks)

Solution

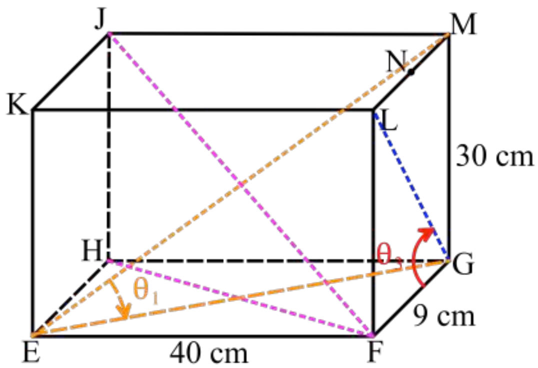
$$FH = \sqrt{40^2 + 9^2} = \sqrt{1681} = 41 \text{ cm}$$

$$FJ = \sqrt{41^2 + 30^2} \quad M_1$$

$$= 50 \cdot 80 \text{ cm} \quad A_1$$

(c) the angle between EM and the plane EFGH; (2 marks)

Solution



$$EG = \sqrt{40^2 + 9^2} = 41 \text{ cm} \quad B_1$$

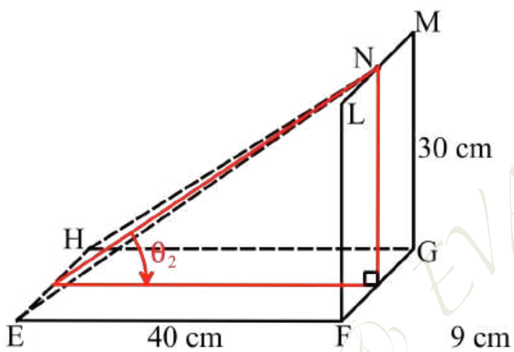
$$\tan \theta_1 = \frac{GM}{EG} = \frac{30}{41}$$

$$\theta_1 = \tan^{-1}\left(\frac{30}{41}\right) \quad M_1$$

$$= 36 \cdot 19^0 \quad A_1$$

(d) the angle between the planes EFGH and ENH; (2 marks)

Solution



$$\tan \theta_2 = \frac{NN'}{O'N'} = \frac{30}{40}$$

$$\theta_2 = \tan^{-1}\left(\frac{30}{40}\right) \quad M_1$$

$$= 36 \cdot 87^0 \quad A_1$$

(e) the angle between the lines EH and GL. (2 marks)

Solution

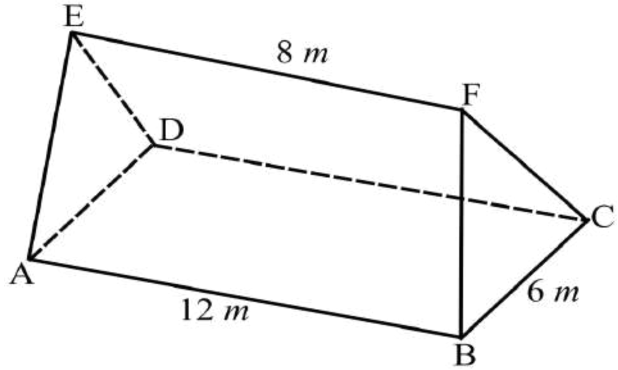
$$\tan \theta_3 = \frac{FL}{FG} = \frac{30}{9}$$

$$\theta_2 = \tan^{-1}\left(\frac{30}{9}\right) \quad M_1$$

$$= 73 \cdot 30^0 \quad A_1$$

27. **2016 paper 2 number 19**

The figure ABCDEF below represents a roof of a house.AB = DC = 12 m,BC = AD = 6 m, AE = BF = CF = DE = 5 cm and EF = 8 m.



(d) Calculate correct to 2 decimal places, the perpendicular distance of EF from the plane ABCD. (3 marks)

Solution

$$\text{Slant height from F to BC} = \sqrt{5^2 - \left(\frac{1}{2}\times 6\right)^2} = 4\text{ m}$$

$$\text{Hence,distance of EF from plane ABCD} = \sqrt{4^2 - 2^2} \qquad M_1$$

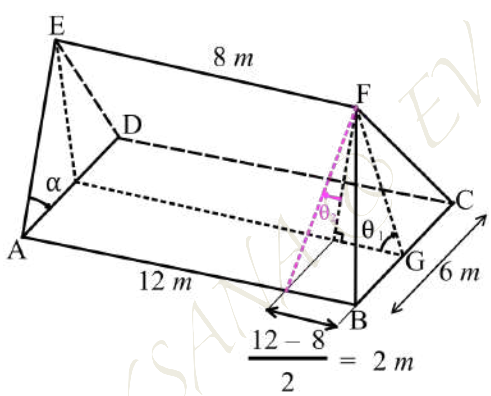
$$= \sqrt{12}$$

$$= 3\cdot 46\text{ m} \qquad A_1$$

(b) Calculate the angle between:

(i) the planes ADE and ABCD; (2 marks)

Solution



$$\tan\theta_1 = \frac{\perp\text{ distance of EF}}{2} = \frac{\sqrt{12}}{2} \qquad M_1$$

$$\theta_1 = \tan^{-1}\left(\frac{\sqrt{12}}{2}\right)$$

$$= 60^0 \qquad A_1$$

(ii) the line AE and the plane ABCD,correct to 1 decimal place; (2 marks)

Solution

$$\sin\alpha = \frac{\perp\text{ distance of EF}}{\text{AE = FB}} = \frac{\sqrt{12}}{5} \text{ or equivalent trig. ratio} \qquad M_1$$

$$\alpha = \sin^{-1}\left(\frac{\sqrt{12}}{5}\right)$$

$$= 43\cdot 9^0 \qquad A_1$$

(iii) the planes ABFE and DCFE,correct to 1 decimal place. (3 marks)

Solution

$$\tan\theta_2 = \frac{\frac{1}{2}\times BC}{\perp \text{ distance of EF}} = \frac{3}{\sqrt{12}} \text{ or equvalent trig. ratio} \quad M_1$$

$$\theta_2 = \tan^{-1}\left(\frac{3}{\sqrt{12}}\right)$$

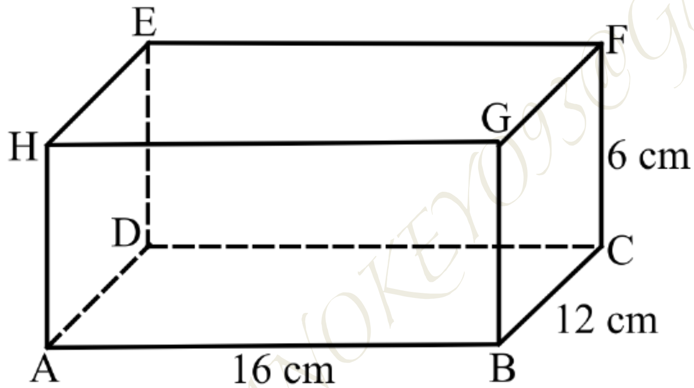
$$= 40\cdot9^0$$

Angle between planeABFE and DCFE $= 2\times 40\cdot 9^0$ $\quad M_1$ for doubling

$$= 81\cdot 8^0 \quad A_1$$

28. 2017 paper 2 number 20.

The figure below represents a cuboid ABCDEFGH in which AB = 16 cm,BC = 12 cm and CF = 6 cm.



(a) Name the projection of the line BE on the plane ABCD. (1 mark)

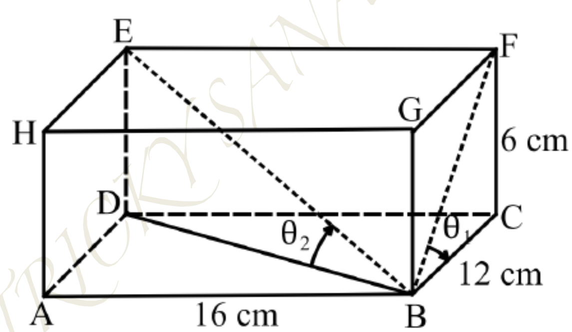
Solution

BD or DB $\quad B_1$

(b) Calculate, correct to 1 decimal place:

(i) the size of the angle between AD and BF; (2 marks)

Solution



Translate line AD to BC

$$\tan\theta_1 = \frac{FC}{BC} = \frac{6}{12}$$

$$\theta_1 = \tan^{-1}\left(\frac{6}{12}\right) \quad M_1$$

$$= 26\cdot 6^0 \quad A_1$$

Accept their equivalents i.e. sine and cosine.

(ii) the angle between line BE and the plane ABCD; (3 marks)

Solution

$$DE = 6 \text{ cm}$$

$$DB = \sqrt{16^2 + 12^2} = \sqrt{400} = 20 \text{ cm} \qquad B_1$$

$$\tan\theta_2 = \frac{DE}{DB} = \frac{6}{20}$$

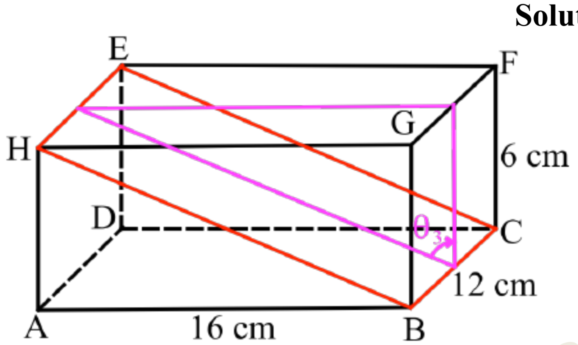
$$\theta_1 = \tan^{-1}\left(\frac{6}{2}\right) \qquad M_1$$

$$= 16\cdot7^0 \qquad A_1$$

Accept their equivalents i.e. sine and cosine.

(iii) the angle between planes HBCE and BCFG. (2 marks)

Solution



Translate line AD to BC

$$\tan\theta_3 = \frac{16}{6}$$

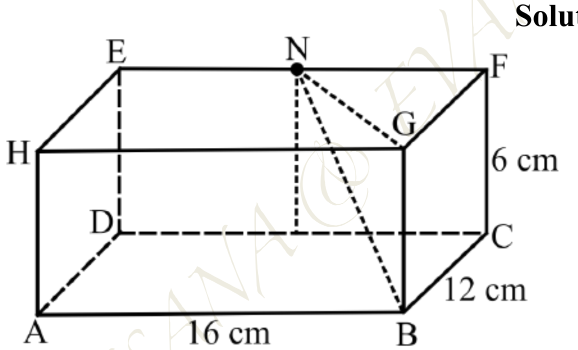
$$\theta_1 = \tan^{-1}\left(\frac{16}{6}\right) \qquad M_1$$

$$= 69\cdot4^0 \qquad A_1$$

Accept their equivalents i.e. sine and cosine.

(c) Point N is the midpoint of EF.Calculate the length of BN,correct to a decimal place. (2 marks)

Solution



$$NG = \sqrt{\left(\frac{1}{2}\times16\right)^2 + 12^2} = \sqrt{208}$$

$$BN = \sqrt{NG^2 + GB^2}$$

$$= \sqrt{\left(\sqrt{208}\right)^2 + 6^2}$$

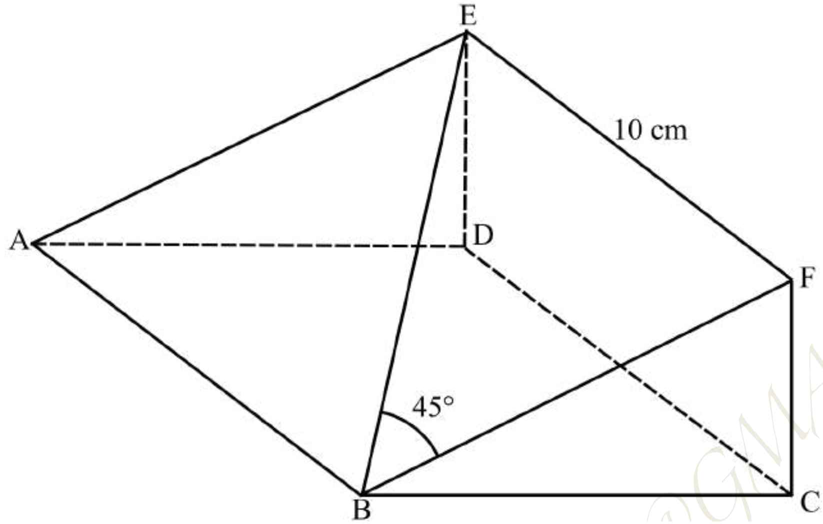
$$= \sqrt{244} \qquad M_1$$

$$= 15\cdot6^0 \qquad A_1$$

Accept their equivalents i.e. sine and cosine.

29. **2018 paper 2 number 4**

The figure below represents a wedge ABCDEF. EF 10 cm, angle FBE 45° and the angle between the planes ABFE and ABCD is 20°.



Calculate length BC, correct to 1 decimal place. (3 marks)

Solution

$BF = 10 \text{ cm}$ B_1 (Isosceles triangle)

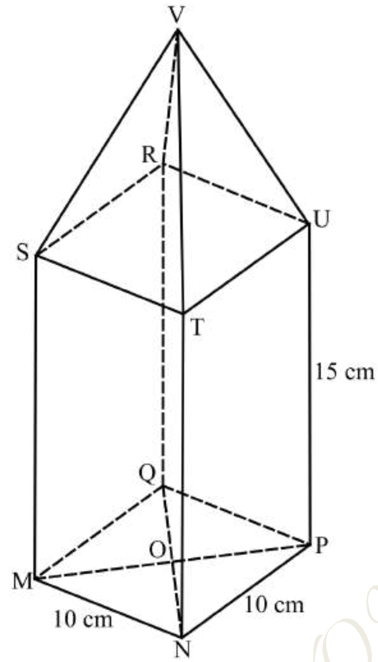
$$\cos 20^\circ = \frac{BC}{BF} = \frac{BC}{10}$$

$BC = 10\cos 20^\circ$ M_1

$= 9{\cdot}4 \text{ cm}$ A_1

30. 2018 paper 2 number 22

The figure below is a model of a watch tower with a square base of sides 10 cm. Height PU is 15 cm and slanting edges UV = TV = SV = RV = 13 cm.



Giving the answer correct to two decimal places, calculate:

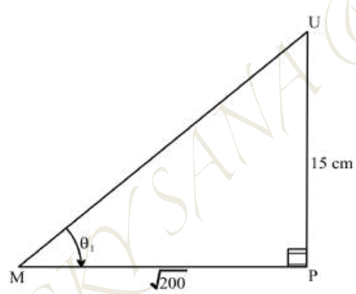
(a) length MP; (2 marks)

Solution

$$MP^2 = MN^2 + NP^2$$
$$MP = \sqrt{10^2 + 10^2}$$
$$= \sqrt{200}$$
$$= 14 \cdot 14 \text{ cm}$$

(b) the angle between MU and plane MNPQ; (2 marks)

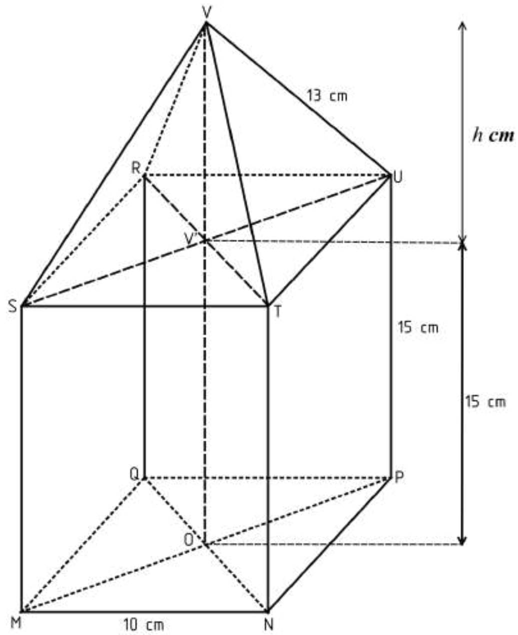
Solution



$$\tan\theta_1 = \frac{15}{\sqrt{200}}$$
$$\theta_1 = \tan^{-1}\left(\frac{15}{\sqrt{200}}\right)$$
$$= 46 \cdot 69^0$$

(c) length of VO; (3 marks)

Solution



$OV = OV' + V'V$

$= 15 + h$

$$h = \sqrt{\left\{13^2 - \left(\frac{\sqrt{10^2+10^2}}{2}\right)^2\right\}} = \sqrt{13^2 - 7\cdot 07^2} = 10\cdot 91$$

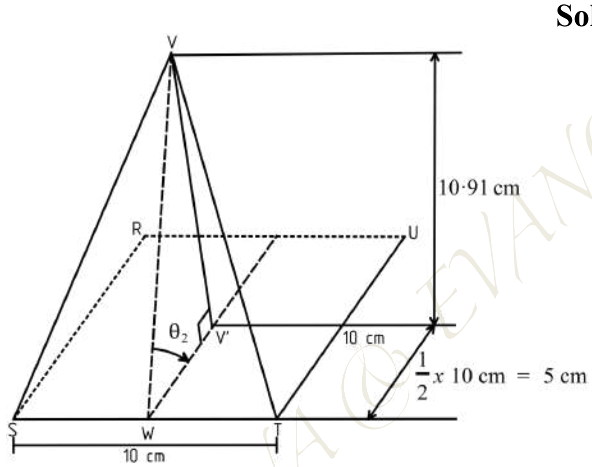
$OV = 15 + h$

$= 15 + 10\cdot 91$

$= 25\cdot 91$ cm

(d) the angle between planes VST and RSTU. (3 marks)

Solution



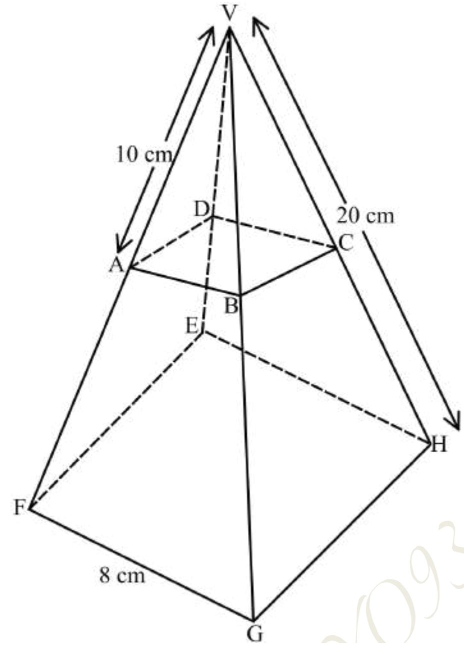
$$\tan\theta_2 = \frac{V'V}{V'W} = \frac{10\cdot 91}{5}$$

$$\theta_2 = \tan^{-1}\left(\frac{10\cdot 91}{5}\right)$$

$= 65\cdot 38^0$

31. 2019 paper 1 number 20.

The figure below is a right pyramid VEFGHI with a square base of 8 cm and a slant edge of 20 cm. Points A,B,C and D lie on the slant edges of the pyramid such that VA = VB = VC = VD = I0 cm and plane ABCD is parallel to the base EFGH.



(a) Find the length of AB. (2 marks)

Solution

By similarity and enlargement;

$$\frac{AB}{8}=\frac{10}{20} \qquad M_1$$

$$AB=\frac{1}{2}\times 8$$

$$=4\text{ cm} \qquad A_1$$

(b) Calculate, correct to 2 decimal places:

(i) The length of AC; (2 marks)

Solution

$$AC=\sqrt{4^2+4^2} \qquad M_1$$

$$=\sqrt{32}$$

$$=5\cdot 66\text{ cm} \qquad A_1$$

(ii) The perpendicular height of the pyramid VABCD (2 marks)

Solution

$$h=\sqrt{\left\{10^2-\left(\frac{1}{2}\times 5\cdot 66\right)\right\}^2}=\sqrt{10^2-\left(2\cdot 83\right)^2} \qquad M_1$$

$$=\sqrt{91\cdot 9911}$$

$$=9\cdot 59\text{ cm} \qquad A_1$$

(c) The pyramid VABCD was cut off. Find the volume of the frustum ABCDEFGH correct to 2 decimal places. (4 marks)

Solution

Volume of frustum ABCDEFGH = Volume of VABCD – Volume of VEFGH

$$= \left\{\frac{1}{3}\times 8\times 8\times\left(2\times 9\cdot 59\right)\right\} - \left\{\frac{1}{3}\times 4\times 4\times 9\cdot 59\right\}$$

$= 409\cdot 17\ cm^3 - 51\cdot 15\ cm^3$

$= 358\cdot 02\ cm^3$

Alternatively;

L.S.F = 1:2

V.S.F = 1:8

Volume of frustum = $V_{Big} - V_{Small} = 7$

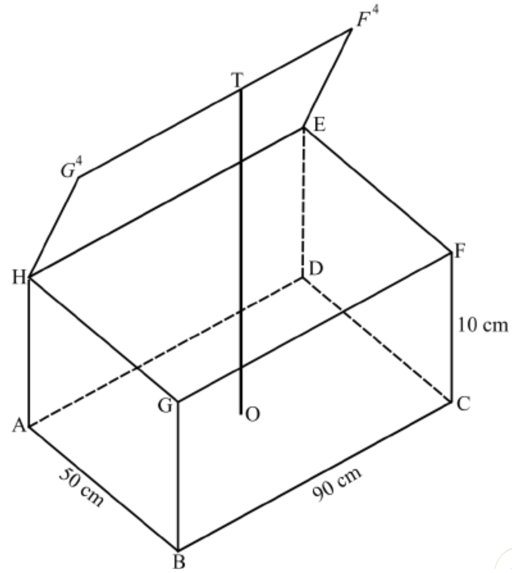
Volume of pyramid = $7\times\ V_{Small}$

$$= 7\times\frac{1}{3}\times 4\times 4\times 9\cdot 59$$

$= 358\cdot 05\ cm^3$

32. 2019 paper 2 number 11.

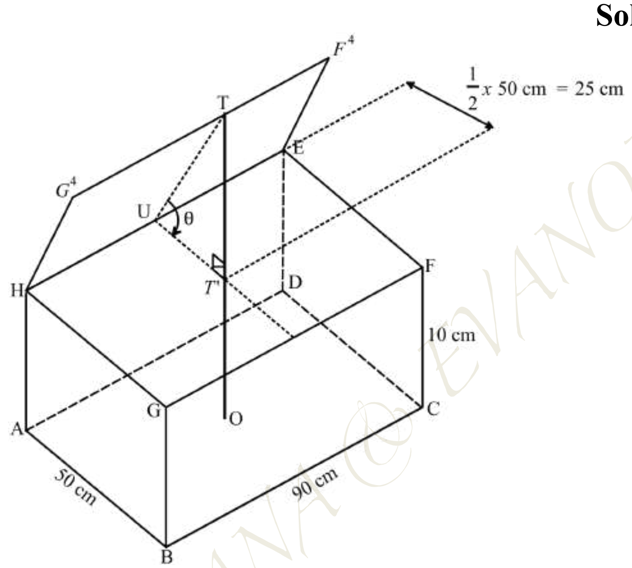
The figure ABCDEFGH represents a box.



The top lid of the box is opened such that the height OT is 35 cm. Calculate the:

(a) angle the top lid makes with the plane FGHE; (2 marks)

Solution



$$\tan\theta = \frac{TT'}{UT} = \frac{25}{25}$$ M_1 or equivalent

$$\theta = \tan^{-1}(1)$$

$$= 45^{\circ}$$ A_1

(b) length BE, correct to 2 decimal places. (2 marks)

Solution

$$BD = \sqrt{(90^2 + 50^2)} = \sqrt{10,600}$$ M_1

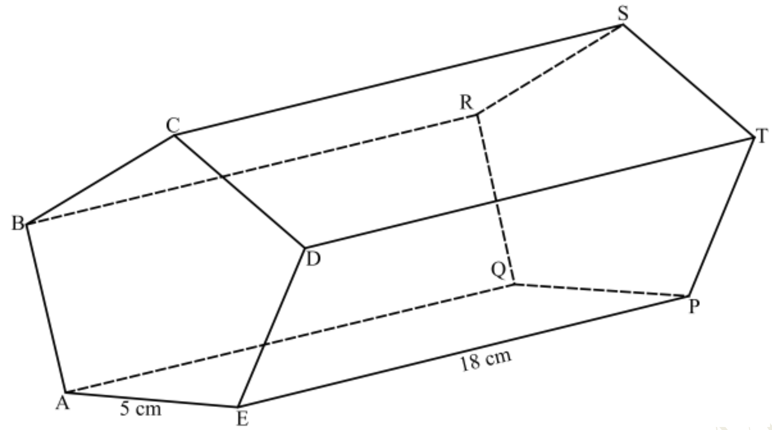
$$BE = \left\{\sqrt{\left(\sqrt{10,600}\right)^2 + 10^2}\right\}$$

$$= \sqrt{10,700}$$

$$= 103 \cdot 44 \text{ cm}$$ A_1

37. Untested model.

The figure below shows a prism whose cross – section is regular pentagon of side 5 cm.The length of the prism is 18 cm.



Calculate the angle between the planes:

(a) AEPQ and ADTQ. (3 marks)

Solution

It is ∠EAD

$$\sin\left(\frac{1}{2}\times\frac{540^0}{5}\right)=\frac{\frac{1}{2}AD}{5}$$

$$\frac{1}{2}AD=5\sin 54^0$$

$$\angle EAD=\cos^{-1}\left(\frac{5\sin 54^0}{5}\right)$$

$$=36^0$$

(b) AEPQ and ACSQ (3 marks)

Solution

It is ∠CAE

$$CE=\sqrt{\left(5^2+5^2\right)-\left(2\times5\times5\times\cos 108^0\right)};\frac{540^0}{5}=108^0$$

$$=8\cdot090169944\text{ cm}$$

$$\cos\left(\angle CAE\right)=\frac{\frac{1}{2}AE}{CE}=\frac{\frac{1}{2}\times5}{8\cdot090169944}=0\cdot309016994$$

$$\angle CAE=\cos^{-1}\left(0\cdot309016994\right)$$

$$=72^0$$

(c) ACSQ and ADTQ (2 marks)

Solution

It is $\angle CAD$

$$AC = AD = \sqrt{(5^2+5^2)-(2\times5\times5\times\cos108^0)};\ \frac{540^0}{5}=108^0$$

$$= 8\cdot090169944 \text{ cm}$$

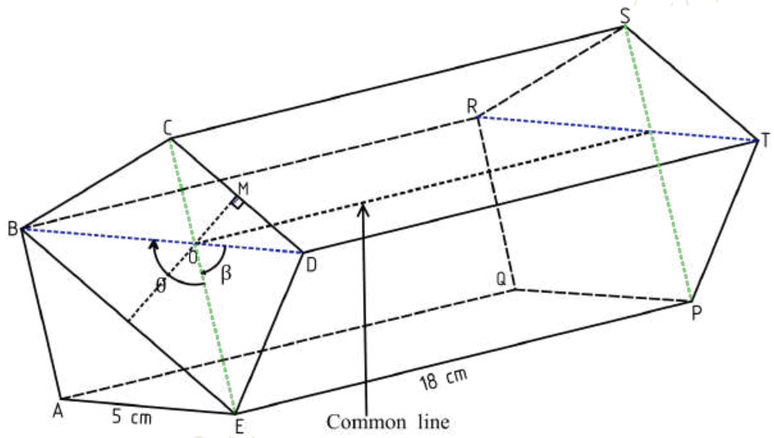
$$\angle CAD = 2\times\sin^{-1}\left(\frac{\frac{1}{2}CD}{AC}\right)=\frac{\frac{1}{2}\times5}{8\cdot090169944}=0\cdot3090169944$$

$$\angle CAD = 2\times\sin^{-1}(0\cdot3090169944)$$

$$=36^0$$

(d) ECSP and BDTR (2 marks)

Solution



Method 1: It is $\angle BOE = \theta$ or $\angle EOD = \beta = 180^0 - \theta$

$$AC = BD = CE = AD = \sqrt{(5^2+5^2)-(2\times5\times5\times\cos108^0)};\ \frac{540^0}{5}=108^0$$

$$= 8\cdot090169944 \text{ cm}$$

$BO = AE = 5$ cm, ABOE is a rhombus

$OD = BD - BO = 8\cdot090169944 - 5 = 3\cdot090169944$ cm

$$MD = \frac{1}{2}CD = \frac{1}{2}\times5 = 2\cdot5 \text{ cm}$$

$$\sin\left(\frac{1}{2}\theta\right)=\frac{MD}{OD}=\frac{2\cdot5}{3\cdot090169944}$$

$$\theta = 2\times\sin^{-1}\left(\frac{2\cdot5}{3\cdot090169944}\right)$$

$$=108^0$$

$$\beta = 180^0-\theta$$

$$=180^0-108^0$$

$$=72^0$$

Method 2:

Since ABOE is a rhombus;

and $\angle BAE = \frac{540^0}{5} = 108^0$ then,

$\angle BOE = \theta = 108^0$ (Opposte interior angle of a rhombus are equal)

$\angle EOD = \beta = 180^0 - \theta$

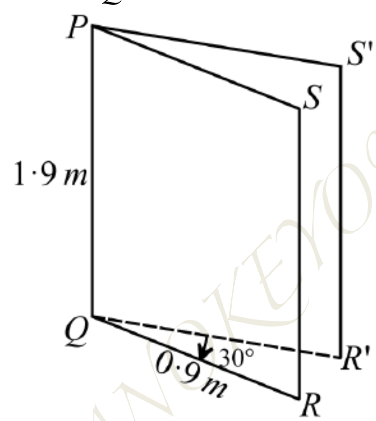
$\beta = 180^0 - \theta$

$= 180^0 - 108^0$

$= 72^0$

38. Untested model 2.

The figure below shows a rectangular door with $PQ = 1 \cdot 9\ m$ and $QR = 0 \cdot 9m$ opened through 30° about the vertical line of hinges PQ to the position $PQR'S'$



Calculate, correct to two decimal places;

(a) the length of SQ ; (2 marks)

Solution

$$SQ = \sqrt{1 \cdot 9^2 + 0 \cdot 9^2}$$
$$= 2 \cdot 10\ m$$

(b) the length of SS'; (2 marks)

Solution

$$SS' = \sqrt{(0 \cdot 9^2 + 0 \cdot 9^2) - (2 \times 0 \cdot 9 \times 0 \cdot 9 \cos 30^0)}$$
$$= 0 \cdot 47\ m$$

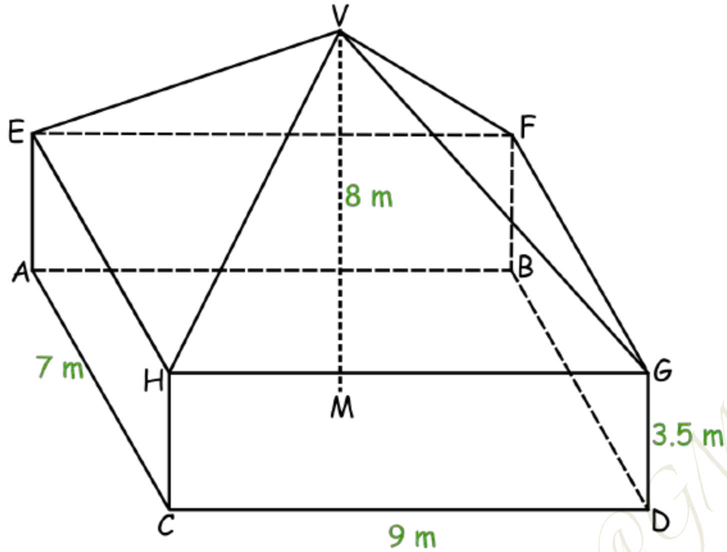
(c) the $\angle SQS'$. (3 marks)

Solution

$$0 \cdot 47^2 = 2 \cdot 10^2 + 2 \cdot 10^2 - (2 \times 2 \cdot 10 \times 2 \cdot 10 \cos\theta)$$
$$-8 \cdot 5991 = -8 \cdot 82 \cos\theta$$
$$\theta = \cos^{-1}\left(\frac{-8 \cdot 5991}{-8 \cdot 82}\right)$$
$$= 12 \cdot 85^0$$

39. Untested model 3.

The diagram below shows a solid made of a cuboid and a pyramid. The apex of the pyramid V is directly above the centre O of ABCD.



Calculate the:

(e) angle between the line DV and the plane ABCD; (2 marks)

Solution

$$MD = \frac{1}{2}AD = \frac{1}{2}\left(\sqrt{9^2+7^2}\right)$$

$$= \frac{1}{2}\times\sqrt{130}$$

$$= 5\cdot 700 \text{ m}$$

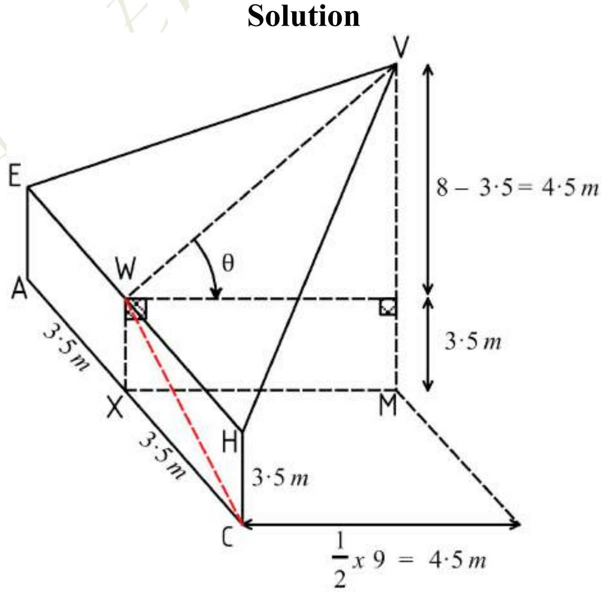
$$\tan(\angle MDV) = \frac{8}{5\cdot 701}$$

$$\angle MDV = \tan^{-1}\theta\left(\frac{8}{5\cdot 700}\right)$$

$$= 54\cdot 53^\circ$$

(f) angle between planes EHV and ACHE. (2 marks)

Solution



Solution

$$\angle VWX = 90^\circ + \tan^{-1}\left(\frac{4\cdot 5}{4\cdot 5}\right)$$

$$= 90^\circ + 45^\circ$$

$$= 135^\circ$$

(g) volume of the pyramid of the solid. (3 marks)

Solution

$$\text{Volume} = \left\{\frac{1}{3}\times 7\times 9\times(8-4\cdot 5)\right\}+\{7\times 9\times 3\cdot 5\}$$

$$= 94\cdot 5 + 220\cdot 5$$

$$= 315\ m^3$$

(h) total surface area of the cuboid and the pyramid. (3 marks)

Solution

$$\text{VG} = \text{VH} = \text{VE} = \text{VF} = \sqrt{5\cdot 701^2 + 4\cdot 5^2} = 7\cdot 263\ m$$

$$\perp h \text{ of face VHG} = \sqrt{7\cdot 263^2 - 4\cdot 5^2} = 5\cdot 701\ m$$

$$\perp h \text{ of face VEH} = \sqrt{7\cdot 263^2 - 3\cdot 5^2} = 6\cdot 364\ m$$

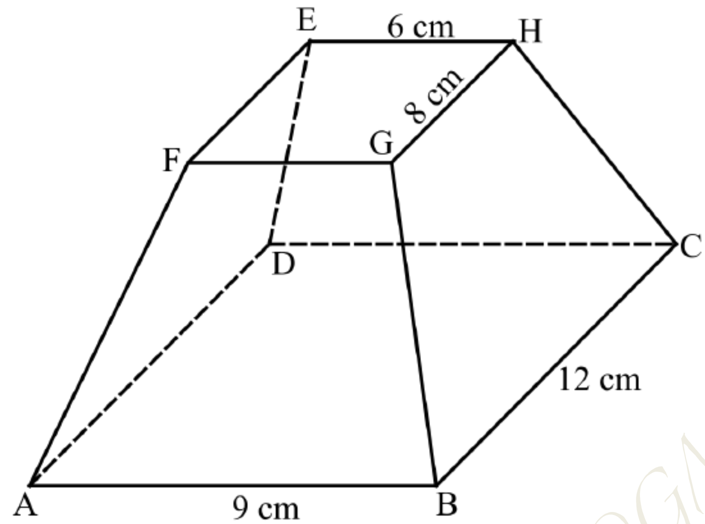
$$\text{S.A} = (7\times 9)+2(7\times 3\cdot 5)+2(9\times 3\cdot 5)+2\left(\frac{1}{2}\times 7\times 6\cdot 364\right)+2\left(\frac{1}{2}\times 9\times 5\cdot 7\right)$$

$$= 63+49+63+44\cdot 548+51\cdot 3$$

$$= 270\cdot 86\ m^2$$

40. Untested model 4.

The figure below a frustum ABCDEFGH a right pyramid.AB = 9 cm,BC = 12 cm,FG = 6 cm,GH = 8 cm and the height of the frustum is 10 cm.



Calculate;

(a) the height of the pyramid ; (2 marks)

Solution

Let the height of chopped of pyramid be x,

$$\text{total height of the pyramd} = 10 + x$$

$$\frac{x}{x+10} = \frac{8}{12}$$

$$x\left(\frac{12}{8} - 1\right) = 10$$

$$x = \frac{10}{0\cdot 5} = 20 \text{ cm}$$

$$\text{Height of the pyramd} = 10 + x = 10 + 20$$

$$= 30 \text{ cm}$$

(d) the length of;

(iii) AC (2 marks)

Solution

$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{9^2 + 12^2} = \sqrt{225}$$

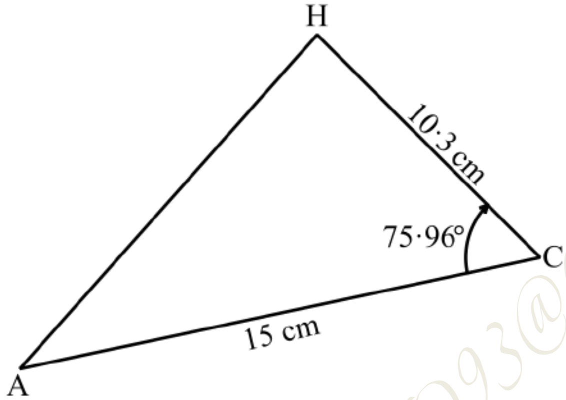
$$= 15 \text{ cm}$$

(iv) AH

Solution

$$HC = \left(\sqrt{30^2 + 7\cdot5^2}\right) - \left(\sqrt{20^2 + 5^2}\right)$$
$$= 30\cdot92 - 20.62$$
$$= 10\cdot3 \text{ cm}$$
$$AC = 15 \text{ cm}$$

$$\angle ACH = \tan^{-1}\left(\frac{30}{7\cdot5}\right)$$
$$= \tan^{-1}(4)$$
$$= 75\cdot96^\circ$$



$$AH = \sqrt{\left(15^2 + 10\cdot3^2\right) - \left(2\times10\cdot3\times15\times\cos 75\cdot96^\circ\right)}$$
$$= \sqrt{256\cdot1268377}$$
$$= 16\cdot00 \text{ cm}$$

(e) the angle between;

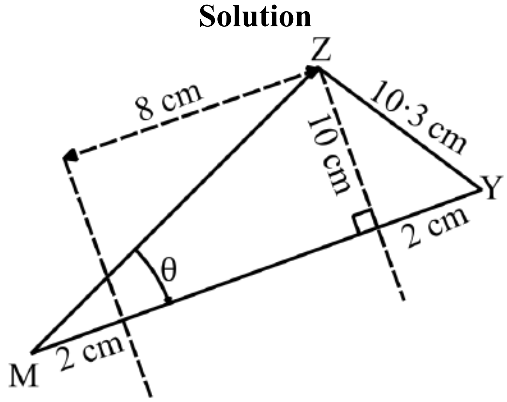
(iii) line AH and the plane ABCD;

Solution

$$\angle HAC = \cos^{-1}\left(\frac{15^2 + 16^2 - 10\cdot3^2}{2\times15\times16}\right)$$
$$= 38\cdot64^\circ$$

(iv) the planes ABHE and ABCD.

Solution



$$\tan\theta = \left(\frac{10}{8+2}\right)$$
$$\theta = \tan^{-1}(1)$$
$$= 45^\circ$$